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The impact of the number of unique preambles on the stability region of the ALOHA algorithm with early feedback

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Introduction: One of the possible ways to increase the throughput of the random multiple access channel in cellular networks is to use preambles with early feedback. On the one hand, an increase in the number of preambles leads to a decrease in the number of collisions, which can improve the throughput. On the other hand, this leads to an increase in the duration of their transmission, which in turn can decrease the throughput. **Purpose:** To study the dependence of the throughput of the random access algorithm of the ALOHA type with an exploration phase on the number of unique preambles. **Results:** We describe a model of a random multiple access system with an ALOHA algorithm using an exploration phase based on early feedback and a limited number of unique preamble. We present a method for calculating the maximum throughput for the system with exact knowledge of the number of active users. We also determine the conditions for maximizing the throughput with different restrictions on the number of unique preambles. Then we propose a procedure for estimating the number of active users for the case when the number of active users is unknown and carry out the analysis of the throughput for such a system. The analysis has shown that the use of the proposed estimation procedure makes it possible to achieve the same throughput with the absence of information on the exact number of active users as with this information provided. **Practical relevance:** The proposed procedure for estimating the number of active users can be used in real-world random multiple access systems with the ALOHA algorithm and the exploration phase. When developing random access systems, the obtained dependence of the throughput on the number of unique preambles allows one to estimate the required number of unique preambles. **Discussion:** Within the framework of the conducted analysis of the throughput, the influence of the number of unique preambles on the duration of their transmission and the influence of this duration on the throughput have not been taken into account, which can be a further research direction.

Keywords – ALOHA, grant-free random access, preamble-based exploration, estimation, throughput, ergodicity, Markov chain.

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Introduction

Long-Term Evolution (LTE) networks use a 4-step random access (RA) scheme, and data transmission is carried out based on scheduling, by allocating resources in the channel based on the preambles received in the first step. The allocation of these resources is called grants, and the RA principle itself is called the grant-based principle [1]. The scenario of massive Machine-Type Communication (mMTC) is characterized by Small Data Transmission (SDT) and the overhead of control signaling becomes significant compared to data transmission [1]. Starting with 3GPP Release 16, it is possible to use a 2-step RA channel in 5G New Radio (NR) [2, 3]. The use of 2-step RA schemes is expected to reduce access latency and increase throughput to support a more number of mMTC devices [4, 5]. The 2-stepRA scheme categorized as grant-free RA.

Regardless of the RA scheme used, the first step involves the selection and transmission of one of the

unique preambles by each user. If the preamble is selected by only one user, then this user will be able to transmit their data without collision. As a rule, preambles are generated based on the Zadoff – Chu sequence. The properties of these sequences are well studied and a detailed description of their properties is considered in [6, 7]. As the number of users increases with a limited number of unique preambles, the average transmission delay in RA mode increases. This is due to the increased probability of a collision occurring during data transmission, which is associated with an increased probability of selecting one preamble by several users. To solve this problem, there is a special mechanism called access class barring (ACB) [8–10]. The idea of this mechanism is that users make a decision to transmit a preamble at the first step of the RA procedure with a certain probability. Thus, not all users will transmit the preamble at step 1 in each iteration of the procedure, which allows reducing the number of collisions and the average transmission delay in RA

systems. To reduce the average delay, the transmission probability in ACB should generally depend on the number of users participating in the RA procedure [11]. Based on the exact knowledge of the number of users ready to transmit, it is possible to select the optimal transmission probability that minimizes the average transmission delay. This strategy uses knowledge of the exact number of users, which is not feasible in practice.

The works [5] and [12] describe and explore the improving of increasing the efficiency of the grant-free RA scheme through early feedback. Early feedback can be achieved by early detection of preamble collisions [13]. Thus, the paper [12] discusses an algorithm based on multi-channel ALOHA, which uses two phases: the Exploration Phase (EP), during which preambles are transmitted, and the Data Transmission Phase (DTP), during which user messages are transmitted. The analysis of this algorithm is given under the assumption that there are no retransmissions, the number of unique preambles is unlimited and the number of independent channels tends to infinity. In addition, it is shown that the maximum throughput per channel can reach $e^{-1}(2 - e^{-1})$.

The paper [14] provides an analysis for the same system and examines the dependence of the maximum throughput per channel on the number of channels. A system with retransmission is also considered and an approach to stabilizing this algorithm with an unlimited number of unique preambles is described. The proposed approach requires knowing exactly how many users have a message ready to be transmitted, which is difficult in real systems. Further, in the paper [15] provides a rule for estimating the number of active users and an analysis of the stability region of such a system. The paper [16] considers a system with one channel without retransmissions and shows the dependence of throughput on the number of unique preambles.

This paper addresses a similar problem, but for a system with retransmissions.

For modern cellular networks, there are various quality of service (QoS) criteria that are analyzed when studying such systems [17]. In this paper, the main QoS criterion when analyzing a 2-step RA scheme is the maximum throughput up to which the system remains stable. There are also various methods for increasing the efficiency of IoT systems, for example, those described in [18].

Since, 5G generation networks are considered as a possible environment for data transmission of computing networks. In this case, an important indicator is the maximum readiness to service emerging requests [19]. When transmitting data in RA mode, various scenarios or types of traffic can be considered. For example, video transmission is a subtype of heterogeneous traffic, for which it is

quite important to reduce the average delay in data delivery [20]. For these cases, it remains important to ensure maximum throughput, which can be achieved by the approaches in the work under consideration.

In this work presents a model of a system, assuming that the number of active user is known, and indicates the differences from the work of [12]. The operation of the system is described by a one-dimensional Markov chain, and it is shown that the problem of calculating throughput is reduced to determining the conditions of ergodicity (stability) of the Markov chain. Besides it, we consider the rejection of the assumption that the number of active users is known, propose a procedure for estimating this number, show that the operation of the system is described by a two-dimensional Markov chain, and propose a method for determining the ergodicity conditions. In addition, we describe the impact of the number of unique preambles and the active user estimation procedure parameter on throughput.

System model

To describe the model, we introduce a number of assumptions.

Assumption 1. There are L unique preambles. A preamble is a sequence of bits of some length that is much shorter than the length of user messages. The duration of the preamble transmission is the same. The base station (BS) always reliably determines the number of unique preambles received from the channel. Information about the number of preambles received is communicated to all users without errors through the feedback channel.

Remark 1: Typically, in RA schemes that use BS preambles, one wants to find all the uniquely received preambles. In our case, as in [12], BS only needs to determine the number of unique preambles received.

Assumption 2. The total operating time of the system is divided into frames of equal length. The frames are numbered by the natural numbers t . Each frame is divided into two phases: the EP with a duration of T_p and the DTP with a duration of T_d . During the EP, each user can choose and transmit one of the unique preambles. During the DTP, users transmit messages. Each phase ends with a response from the BS. Users know the exact frame boundaries, as well as phases, and can participate in the RA procedure only at the beginning of each frame.

Assumption 3. At each of the phases, one of the following events may occur:

- success – preamble or data was transmitted by a single user;
- empty – no data or preamble transmitted;

– conflict – preambles or data were transmitted by two or more users simultaneously.

Remark 2: If there is a conflict arises at the EP, then in accordance with assumption 1, the BS will reliably determine the number of unique preambles received. If a conflict arises during the DTP, then user messages cannot be correctly received on the BS.

Assumption 4. The system has a potentially infinite number of users. In this case, the concepts of “user message” and “user” are equivalent. The number of messages entering into the system in a single frame is distributed according to the Poisson’s law with the parameter λ .

Assumption 5. The case is considered when $T_p \ll T_d$, so the following is assumed: $T_p = 0$ time units, and $T_d = 1$ time units.

Assumption 6. The number of active users in frame number t is known. Let’s denote it as M_t .

The operation of the system under consideration can be described using the following algorithm.

Step 1. Each user decides to transmit the preamble with the following probability:

$$P_{EP} = \min\left(1, \frac{G}{M_t}\right), \quad (1)$$

where G is a parameter that affects the probability of transmission to the EP.

Step 2. Users who decide to transmit during the EP randomly select one of the L unique preambles.

Step 3. The BS determines the number of unique preambles received. Transmits information about this number to all users through the feedback channel.

Step 4. If there is only one preamble in the channel, then the user who passed that preamble sends their data with one hundred percent probability during the DTP. If more than one preamble is detected, it is up to users to independently decide whether to transmit their data during the DTP with probability

$$P_{DTP} = \frac{1}{U},$$

where U is the number of unique preambles detected during the EP.

Step 5. The BS informs all users about event occurred in the frame through the feedback channel. If the user who transmitted receives information that his message has been successfully transmitted, then he leaves the system. Otherwise, the user repeats the previous steps of the algorithm until their message is successfully transmitted.

Steps 1–3 define the EP, while steps 4 and 5 define the DTP.

A similar system model was considered in [12]. Assumptions 2–5 of the presented system model are

fully consistent with the assumptions from [12]. The main differences between the system discussed in [12] and the one presented here are follows:

– the presence of an unlimited number of unique preambles (assumption 1 differs in that the number of unique preambles is finite and equal to L);

– the presence of a large number of independent channels (only one channel is considered in this work);

– at step 1 of the algorithm, all users transmit with probability 1 [in the algorithm under consideration, the probability is determined by equation (1)];

– there are no retransmissions and a user who made the transmission during DTP leaves the system regardless of the event in the channel (there are retransmissions in this work, see step 5 of the algorithm).

Throughput with the knowledge of the exact number of active users

The number of active users in the system under consideration can be described by the following recurrent expression:

$$M_{t+1} = M_t - N_t + V_t,$$

where M_t and M_{t+1} are the number of active users in the system in frames t and $t + 1$; N_t is the number of users leaving the system in the frame t ; V_t is the number of users entering the system during the frame t , distributed according to the Poisson’s law with the parameter λ .

Thus, the sequence of random variables M_t defines a homogeneous irreducible aperiodic Markov chain. Based on the results from [15], we will understand the throughput as the upper bound of the input arrival rate, up to which the Markov chain is ergodic:

$$T(G, L) = \sup_{\lambda} \{\lambda : \text{Markov chain ergodic}\}.$$

Using approaches from the works [21], it can be shown that the value of $T(G, L)$ can be calculated using the following equation:

$$T(G, L) = \lim_{n \rightarrow \infty} \Pr\{B_t | M_t = n\}. \quad (2)$$

Let us show how to calculate the probability $\Pr\{B_t | M_t = n\}$. Based on the definition given earlier, this probability can be calculated as follows:

$$\Pr\{B_t | M_t = n\} = n p_{EP} (1 - p_{EP})^{n-1} + \sum_{i=2}^n C_n^i p_{EP}^i (1 - p_{EP})^{n-1} \Pr\{B_t | i\},$$

where $\Pr\{B_t|i\}$ is the probability of the event “Success” for i users and L preambles. This probability can be calculated using the formula

$$\Pr\{B_t|i\} = \sum_{j=2}^{\min(i,L)} \Pr\{j|i\} \Pr\{B_t|j, i\}. \quad (3)$$

Formula (3) consists of the product of two probabilities: $\Pr\{j|i\}$ is the probability that i users will independently select and transmit j unique preambles from a total of L unique preambles; $\Pr\{B_t|j, i\}$ is the probability of the “Success” event with j preambles detected after the EP and i active users.

The probability $\Pr\{j|i\}$ is reduced to the following combinatorial problem: there are L boxes, and i indistinguishable objects distributed randomly among these boxes. It is necessary to calculate the probability that j boxes will contain at least one object.

The book [22] describes a well-known combinatorial problem in which the probability of the distribution of empty boxes is found [p. 108, formula (2.4)]. A similar formula was re-derived in the work [8] [formula (19)]. Using the result from [22, 8] and modifying it for our problem, the desired probability can be calculated as follows:

$$\Pr\{j|i\} = C_L^{L-j} \sum_{v=0}^j (-1)^v C_j^v \left(1 - \frac{L-j+v}{L}\right)^i. \quad (4)$$

The probability $\Pr\{B_t|j, i\}$ can be calculated as

$$\Pr\{B_t|j, i\} = i \frac{1}{j} \left(1 - \frac{1}{j}\right)^{i-1}. \quad (5)$$

Generalizing formulas (3)–(5) and substituting the value $p_{EP} = \frac{G}{M_t}$, formula (2) will have the following form:

$$\begin{aligned} T(G, L) = \lim_{n \rightarrow \infty} \Pr\{B_t | M_t = n\} &= Ge^{-G} + \\ &+ \sum_{i=2}^{\infty} \frac{G^i}{i!} e^{-G} \sum_{j=2}^{\min(i,L)} C_L^{L-j} \sum_{v=0}^j (-1)^v \times \\ &\times C_j^v \left(1 - \frac{L-j+v}{L}\right)^i i \frac{1}{j} \left(1 - \frac{1}{j}\right)^{i-1}. \end{aligned}$$

Based on equation (2), it is clear that for a fixed number of unique preambles, the value of G affects the value of throughput. The function is monotone and unimodal for all positive values of G on the interval from 0 to infinity. Consequently, the throughput value for a given number of preambles can be maximized by the parameter G . We will denote the maximum throughput value for a given number of preambles L as $T_{\max}(L) = \max_G T(G, L)$, and

■ Maximum throughput and the corresponding parameter values G and Q

L	$T_{\max}(L)$	$G_{\text{opt}}(L)$	$Q_{\text{opt}}(L)$
1	0.367	1.00	0.0000
2	0.448	1.31	0.1360
4	0.498	1.54	0.2150
8	0.523	1.67	0.3315
16	0.536	1.72	0.4100
32	0.542	1.74	0.4470
64	0.545	1.75	0.4683
∞	0.548	1.77	0.4917

the value of the parameter at which it is achieved as $G_{\text{opt}}(L) = \arg \max_G T(G, L)$. The results of the calculation of these values for different numbers of preambles are shown in Table (see columns 2 and 3).

Remark 3: If we consider the presence of an unlimited number of unique preambles, then the calculation of the throughput will be performed according to the following formula [14]:

$$\lim_{n \rightarrow \infty} T(G, L) = Ge^{-G} + \sum_{i=2}^{\infty} \left(1 - \frac{1}{i}\right)^{i-1} \frac{G^i}{i!} e^{-G}.$$

The calculation result for this case is given in the last row of the Table.

It should be noted that at $L = 1$ the algorithm in question corresponds to the regular ALOHA algorithm, and the maximum throughput is reached at $G = 1$. With an increase in the number of preambles, the optimal value of G increases, which means an increase in the probability of transmission during the EP.

Throughput when using the procedure for estimating the number of active users

In real-world systems, there is no way to reliably know the number of active users at the beginning of each frame to determine the probability of transmission during the EP. To approximate real conditions, one can use some procedure for estimating the number of active users in a frame based on the approach proposed in [21]. In this case, assumption 6 is excluded from the system of assumptions. Step 1 of the system operation algorithm is modified as follows.

1. First, the procedure for estimating the number of active users is performed according to the following recurrent rule (where $S_1 = 1$):

$$S_{t+1} = \max[1, S_t + aI\{A_t\} + bI\{B_t\} + cI\{C_t\}],$$

where $I\{\cdot\}$ is the indicator function. The coefficients a , b and c are defined as follows:

$$a = Q - 1; b = Q - 1; c = \frac{2}{(e - 2)} + Q,$$

where Q is an estimation parameter that depends on the number of unique preambles.

2. Then the probability of transmission during the EP in frame t is calculated using the equation:

$$P_{EP} = \min\left(1, \frac{G}{S_t}\right). \quad (6)$$

In this case, the operation of this system will be described by a two-dimensional Markov chain (M_t, S_t) .

To determine the ergodicity conditions of a given Markov chain, we calculate the average drift for each component of the given Markov chain:

$$\begin{aligned} E[M_{t+1} - M_t | S_t = s, M_t = n] &= \\ &= \lambda - \Pr\{B_t | S_t = s, M_t = n\}; \\ E[S_{t+1} - S_t | S_t = s, M_t = n] &= \\ = c + (a - c)\Pr\{A_t | S_t = s, M_t = n\} + \\ &+ (b - c)\Pr\{B_t | S_t = s, M_t = n\}, \end{aligned}$$

where $\Pr\{B_t | S_t = s, M_t = n\}$ is the probability of the event "Success" on the DTP and $\Pr\{A_t | S_t = s, M_t = n\}$ is the probability of the event "Empty" on the DTP.

In further derivation we will take into account the fact that the number of users who decided to transmit to the EP is distributed according to the binomial law with parameters p_{EP} and n .

Note that "Success" on the DTP can occur in two situations:

- 1) the "Success" event on the EP;
- 2) the "Conflict" event on the EP and the "Success" event on the DTP.

Taking into account the above, the probability $\Pr\{B_t | S_t = s, M_t = n\}$ can be calculated using the following formula:

$$\begin{aligned} \Pr\{B_t | S_t = s, M_t = n\} &= np_{EP}(1 - p_{EP})^{n-1} + \\ &+ \sum_{i=2}^n C_n^i p_{EP}^i (1 - p_{EP})^{n-i} \Pr\{B_t | i\} = \\ &= \frac{nG}{s} \left(1 - \frac{G}{s}\right)^{n-1} + \sum_{i=2}^n C_n^i \left(\frac{G}{s}\right)^i \left(1 - \frac{G}{s}\right)^{n-i} \times \\ &\times \sum_{j=2}^{\min(i,L)} C_L^{L-j} \sum_{v=0}^j (-1)^v C_j^v \left(1 - \frac{L-j+v}{L}\right)^i \frac{1}{j} \left(1 - \frac{1}{j}\right)^{i-1}. \end{aligned}$$

Similarly, the event "Empty" on the DTP can occur in two situations:

1) the event "Conflict" on the EP and the event "Empty" on the DTP;

2) the event "Empty" on the EP.

Therefore, the probability $\Pr\{A_t | S_t = s, M_t = n\}$ is determined as follows:

$$\begin{aligned} \Pr\{A_t | S_t = s, M_t = n\} &= (1 - p_{EP})^n + \\ &+ \sum_{i=2}^n C_n^i p_{EP}^i (1 - p_{EP})^{n-i} \Pr\{A_t | i\} = \\ &= \left(1 - \frac{G}{s}\right)^{n-1} + \sum_{i=2}^n C_n^i \left(\frac{G}{s}\right)^i \left(1 - \frac{G}{s}\right)^{n-i} \times \\ &\times \sum_{j=2}^{\min(i,L)} C_L^{L-j} \sum_{v=0}^j (-1)^v C_j^v \left(1 - \frac{L-j+v}{L}\right)^i \left(1 - \frac{1}{j}\right)^i. \end{aligned}$$

As was done in paper [21] on determining the conditions of ergodicity, we will introduce the functions $\mu_n(\lambda, k)$ and $\mu_s(k)$:

$$\begin{aligned} \mu_n(\lambda, k) &= \lambda - \lim_{\substack{n \rightarrow \infty \\ s \rightarrow \infty \\ k = n/s}} \Pr\{B_t | S_t = s, M_t = n\}; \\ \mu_s(k) &= c + (a - c) \lim_{\substack{n \rightarrow \infty \\ s \rightarrow \infty \\ k = n/s}} \Pr\{A_t | S_t = s, M_t = n\} + \\ &+ (b - c) \lim_{\substack{n \rightarrow \infty \\ s \rightarrow \infty \\ k = n/s}} \Pr\{B_t | S_t = s, M_t = n\}. \end{aligned}$$

Taking into account that when $n \rightarrow \infty$ and $s \rightarrow \infty$ the binomial distribution turns into a Poisson distribution with the parameter $k = n/s$, therefore the limits indicated above can be calculated as follows:

$$\begin{aligned} \lim_{\substack{n \rightarrow \infty \\ s \rightarrow \infty \\ k = n/s}} \Pr\{B_t | S_t = s, M_t = n\} &= \\ &= Gke^{-Gk} + \sum_{i=2}^{\infty} e^{-Gk} \frac{(Gk)^i}{i!} \Pr\{B_t | i\} = \\ &= e^{-Gk} \left(Gk + \sum_{i=2}^{\infty} \frac{(Gk)^i}{i!} \sum_{j=2}^{\min(i,L)} \frac{i}{j} \left(1 - \frac{1}{j}\right)^{i-1} \Pr\{j | i\} \right); \\ \lim_{\substack{n \rightarrow \infty \\ s \rightarrow \infty \\ k = n/s}} \Pr\{A_t | S_t = s, M_t = n\} &= \\ &= e^{-Gk} + \sum_{i=2}^{\infty} e^{-Gk} \frac{(Gk)^i}{i!} \Pr\{A_t | i\} = \\ &= e^{-Gk} \left(1 + \sum_{i=2}^{\infty} \frac{(Gk)^i}{i!} \sum_{j=2}^{\min(i,L)} \left(1 - \frac{1}{j}\right)^i \Pr\{j | i\} \right). \end{aligned}$$

Therefore:

$$\begin{aligned} \mu_n(\lambda, k) &= \lambda - e^{-Gk} \left(Gk + \sum_{i=2}^{\infty} \frac{(Gk)^i}{i!} \times \right. \\ &\quad \left. \times \sum_{j=2}^{\min(i,L)} \frac{i}{j} \left(1 - \frac{1}{j}\right)^{i-1} \Pr\{j|i\} \right); \\ \mu_s(k) &= c + (a-c)e^{-Gk} \left(1 + \sum_{i=2}^{\infty} \frac{(Gk)^i}{i!} \times \right. \\ &\quad \left. \times \sum_{j=2}^{\min(i,L)} \left(1 - \frac{1}{j}\right)^i \Pr\{j|i\} \right) + \\ &+ (b-c)e^{-Gk} \left(Gk + \sum_{i=2}^{\infty} \frac{(Gk)^i}{i!} \sum_{j=2}^{\min(i,L)} \frac{i}{j} \left(1 - \frac{1}{j}\right)^{i-1} \Pr\{j|i\} \right). \end{aligned}$$

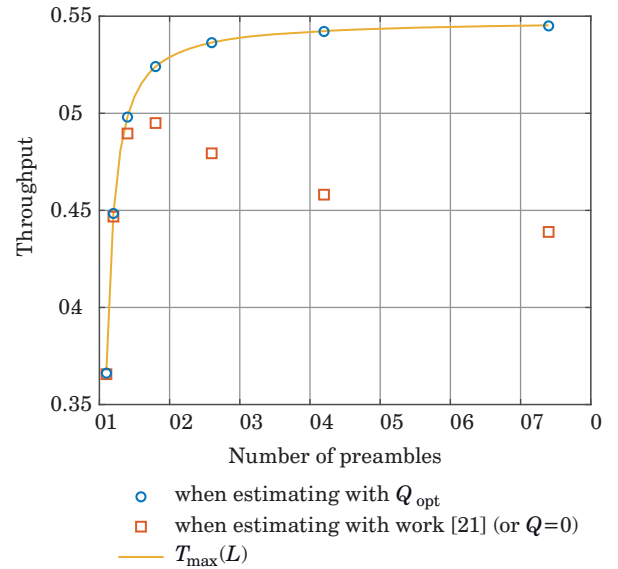
It follows from the work [21] that in order to determine whether the Markov chain (M_t, S_t) is ergodic for some value of λ , the following steps must be taken:

- 1) for a fixed value of λ , solve the equation $\mu_n(\lambda, k) = k\mu_s(k)$ for the variable k ;
- 2) check that the following condition is met for all roots of the equation: $\mu_n(\lambda, k_i) < 0$; $\mu_s(k_i) < 0$;
- 3) if the condition is satisfied, then the Markov chain is ergodic.

For a fixed a given number of unique preambles L and throughput with exact knowledge of the number of active users $T(G, L)$, we can numerically find the estimation parameter Q for which, according to the estimation procedure, the Markov chain is ergodic. For a throughput equal to $T_{\max}(L)$, the value of the parameter Q , at which the Markov chain is ergodic, will be called optimal and denoted as $Q_{\text{opt}}(L)$. The values of $Q_{\text{opt}}(L)$ for a different numbers of unique preambles are also presented in Table (see column 4).

Impact of the estimation procedure parameter on throughput

The work [21] describes a similar procedure for estimating the average number of active users for a typical ALOHA algorithm with a single channel. The difference between the estimation procedure proposed in this work and [21] is the introduction of the parameter Q . It can be noted that the Markov chain which describes the operation of our system for $L = 1$ and $G = 1$ completely coincides with the Markov chain from the paper [21]. In this case, for the system in question $Q_{\text{opt}} = 0$, so the estimation procedure will be exactly the same as described in [21]. If we use the estimation procedure for our system with $Q = 0$ (the estima-



■ Throughput versus number of unique preambles

tion procedure is similar to [21]), then for a given number of unique preambles, the throughput value will not reach the possible maximum. To maximize throughput, one must use the values of $Q_{\text{opt}}(L)$. Based on these results, we can conclude that using $Q_{\text{opt}}(L)$ one can achieve a throughput value of $T_{\max}(L)$ when knowing the exact number of active users. These results are shown in Figure. The values of $T_{\max}(L)$ were calculated according to with the definition given earlier. The throughput values based on our estimation were obtained for some values of L taking into account the ergodicity of the Markov chain.

Conclusion

This paper discusses a system with retransmissions, a single channel, and an ALOHA algorithm with an exploration phase with a limited number of unique preambles. The calculation of throughput and its maximization for a different number of unique preambles with a known number of active users is given. A modification of the algorithm considered in the work is described, which includes a procedure for estimating the number of active users. It has been shown that by estimating with this procedure the number of active users, it is possible to achieve the same throughput as if the number of these users were known exactly.

If the number of unique preambles is 32 or more, the throughput values become close to the value in the case of throughput, when the number of preambles is infinite (see Table). In addition, it can be assumed that by increasing the num-

ber of channels used in the system with a limited number of preambles, it is possible to achieve a throughput per channel equal to $e^{-1}(2 - e^{-1})$ as shown in paper [12] for a system without retransmissions and an unlimited number of unique preambles.

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Влияние числа уникальных преамбул на область стабильной работы алгоритма АЛОХА с ранней обратной связью

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Введение: одним из возможных путей повышения пропускной способности канала случайного множественного доступа в со-товых сетях является использование преамбул с ранней обратной связью. Увеличение числа преамбул приводит, с одной стороны, к уменьшению числа коллизий, что может повысить пропускную способность, с другой стороны — к увеличению длительности их передачи, что, в свою очередь, может понизить пропускную способность. **Цель:** исследовать зависимость пропускной способности алгоритма случайного доступа типа АЛОХА с фазой исследования от числа уникальных преамбул. **Результаты:** описана модель системы случайного множественного доступа с алгоритмом типа АЛОХА при использовании фазы исследования на основе ранней обратной связи и ограниченным числом уникальных преамбул. Представлен способ вычисления максимальной пропускной способности для системы при точном знании числа активных абонентов. Определены условия для максимизации пропускной способности при различном ограничении на число уникальных преамбул. Предложена процедура оценки числа активных абонентов для случая, когда число активных абонентов неизвестно. Для системы с неизвестным числом активных абонентов проведен анализ пропускной способности. В результате анализа показано, что при отсутствии информации о точном количестве активных абонентов использование предложенной процедуры оценки позволяет добиться такой же пропускной способности, как и при наличии этой информации. **Практическая значимость:** предложенная процедура оценки числа активных абонентов может быть использована в реальных системах случайного множественного доступа с алгоритмом АЛОХА и фазой исследования. При разработке систем случайного доступа полученная зависимость пропускной способности от числа уникальных преамбул позволяет оценить необходимое количество уникальных преамбул. **Обсуждение:** в рамках проведенного анализа пропускной способности не учитывалось влияние числа уникальных преамбул на длительность их передачи и влияние этой длительности на пропускную способность, что может быть дальнейшим направлением исследования.

Ключевые слова — АЛОХА, случайный доступ без выделения грантов, фаза исследования на основе преамбул, оценка, пропускная способность, эргодичность, марковская цепь.

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