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Articles

Comparative analysis of ALOHA based algorithms with early feedback

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Introduction: *One option for increasing the throughput of a random multiple access system is to use early feedback. Early feedback refers to a rapid response from the base station after receiving the preambles. In this paper, early feedback is considered for multiple access systems using different ALOHA-based algorithms.* ***Purpose:*** *To conduct a comparative analysis of the dependence of the maximum throughput on the number of unique preambles of random access algorithms based on the ALOHA algorithm with early feedback.* ***Results:*** *The paper considers the ALOHA algorithm with an exploration phase, the 2-step ALOHA algorithm and a combination of these two algorithms. For these three algorithms we carry out a comparative analysis for the variant with a known number of unique users and the variant with an estimation of the number of active users. The study shows that the use of the procedure for estimating the number of active users allows achieving similar values of the dependence of the maximum throughput on the number of unique preambles as for the first variant with a known number of active users. In addition, it is shown that the use of a fixed parameter affecting the estimation procedure, with the value of this parameter equal to its optimal for infinite numbers of preambles, leads to a loss that does not exceed 6% for any numbers of preambles, and does not exceed 0.2% for 30 or more preambles.* ***Practical relevance:*** *New algorithm based on ALOHA with early feedback is proposed, which allows increasing the maximum throughput of the system as compared to previously known algorithms. This algorithm can be used in the random access channel of future generation networks.* ***Discussion:*** *The analysis does not take into account the influence of the number of channels used in the system, which could be a further direction of research.*

Keywords *— ALOHA, grant-free random access, preamble-based exploration, estimation, throughput, ergodicity, Markov chain.*

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Introduction

The random access (RA) procedure in 4G (LTE) networks is implemented using a 4-step scheme [1, 2]. Its mechanism is as follows: first, the user selects and sends a unique preamble. The base station then attempts to decode the preambles and sends a response allocating resources for data transmission. Then the user uses this resource to send his data. A collision occurs when multiple users select the same preamble. At the final stage, the base station informs all users about an event in the channel after frame of RA procedure via feedback channel. The distribution of these resources is called grants, and the RA principle itself is called the grant-based principle [3].The main disadvantage of this scheme becomes apparent when a lot of devices are connected: the number of collisions increases sharply and, as a consequence, the time it takes to resolve them increases, which causes significant delays.

To solve this problem, the transmission decision in the first stage is applied randomly by each user, which helps to reduce the number of collisions, this mechanism is called access class barring (ACB) [4, 5]. The essence of this mechanism is to apply a probabilistic approach to the transmission of the preamble at the first stage of the RA procedure. By allowing users to transmit the preamble not in every iteration, but only with a given probability, the total number of collisions and average delay are reduced. To minimize latency in the ACB method, the transmission probability should be related to the number of users attempting to gain access [6]. With precise knowledge of this quantity allows you to select the optimal probability. However, in practice, determining the exact number of users is a non-trivial task.

The development of modern wireless communication networks is actively linked to the development of the Internet of Things (IoT) technology. Within this technology, one of the key scenarios is massive Machine-Type Communication (mMTC), which is characterized by the operation of a large number of user devices, to which different requirements apply, which affects the operation of networks. The use of the 4-step scheme in the conditions of mMTC is characterized by two main disadvantages: a high ratio of service traffic to payload and an increase in transmission time due to delayed collision detection. In 5G NR, an alternative 2-step procedure has been introduced [7, 8], the key feature of which is the sending of a single combined message (preamble and user data) in the first step. This reduces the number of signaling messages and the required base station computing resources. The 2-step RA scheme is classified as a grant-free random access procedure.

The random access schemes described begin with the selection and transmission of one of the unique preambles to users. Typically, preambles are generated based on the Zadoff — Chu sequence. The properties of these sequences are well studied, and their detailed description is discussed in [9, 10].

In [11] and [12], the efficiency improvement of the grant-free RA scheme by means of early feedback is described and investigated. Early feedback can be achieved by early detection of preamble collisions [13]. Thus, in the paper [12] an algorithm based on the multi-channel ALOHA algorithm is discussed, which uses two phases: the exploration phase (EP), during which preambles are transmitted, and the data transmission phase (DTP), during which user messages are transmitted. The analysis of this algorithm is carried out under the assumption that there are no retransmissions, the number of unique preambles is unlimited, and the number of independent channels tends to infinity. In addition, it is shown that the maximum throughput per channel can reach $e^{-1}\left(2-e^{-1}\right)$.

The paper [14] presents an analysis of the same system and investigates the dependence of the maximum channel throughput on the number of unique preambles. A system with retransmission is also considered and an approach to stabilizing this algorithm with a limited number of unique preambles and a single channel is described. Also, in [14] a system is analyzed that uses a specific approach to estimate the number of active users. In the paper [15] a system with one channel and repeated transmissions is considered. For such a system, a 2-step ALOHA algorithm using an early feedback approach is analyzed. In addition, for this algorithm shows the dependence of throughput on the number of unique preambles.

Increasing the throughput for the random multiple access procedure and the use of switching with duplicate routes [16, 17] together can lead to an increase in the probability of message delivery in the system.

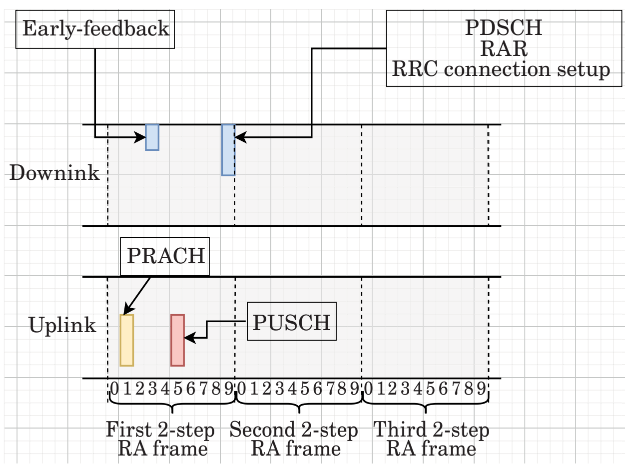
This paper describes a combined random multiple access algorithm that allows increasing the throughput compared to [12], but with a smaller number of channels in the system. Taking into account the use of an estimation procedure similar to that in [14], it is shown that the functioning of the system is described by a two-dimensional Markov chain. The use of this approach allows us to eliminate the necessary condition of an exact number of active users at the beginning of each frame of the RA procedure. In addition, a comparison is made of the dependence of the throughput on the number of unique preambles and the parameter of the procedure for estimating the number of active users for three different algorithms.

Description of the early feedback approach

The idea of early feedback

As was written earlier in the work [12], an approach called early feedback is defined. This approach implies a quick response from the base station after receiving preambles from all users and before users begin transmitting their data. Thus, it is important to note that the frame duration of the RA procedure remains unchanged. Let us consider the application of this approach to a 2-step RA procedure. In this case, RA frame consists of transmitting two messages: message A and message B. Message A represents the transmission of the preamble and the user data (yellow and red blocks shown in Fig. 1, respectively). Message B is a combined response for the received preamble and data (the second blue block shown in Fig. 1). An example of a time-frequency diagram for this approach is shown in Fig. 1.

In addition, in the work [12] the author considers the application of the early feedback approach to the multi-channel RA algorithm ALOHA with an infinite number of channels and an infinite number of preambles.



■ ***Fig. 1.*** Time-frequency diagram of a 2-step RA procedure with early feedback

Using early feedback for the ALOHA-based algorithms

In [14], a modification of the algorithm from [12] is considered for a system with one channel, a limited number of unique preambles and repeated transmissions (users leave the system only after successful transmission of their data). The modified algorithm's frame consists of two phases: an EP and a DTP. During the exploration phase, users transmit preambles, and during the data transmission phase, users transmit their's data.

In [15], the authors consider the application of the early feedback approach to the 2-step ALOHA algorithm. This algorithm is considered for a system with 1 channel, a limited number of preambles and the presence of repeated transmissions. It is important to note that in this case the base station does not need to determine the exact number of preambles received, so simpler preambles can be used.

In this paper, we will consider a combined RA algorithm based on ALOHA with a exploration phase. This combination consists of using a EP similar to the algorithm and the DTP of the algorithm from [14], as well as changing the operation of the RA procedure in the case of the "Empty" event after the EP similar to the algorithm from [15].

The operation of system for these algorithms presented in the Table 1.

In the Table 1 we introduce G — an algorithm parameter that affects the transmission probability during the EP; M_t — the exact number of active users at the beginning of the RA algorithm frame; U — the number of preambles received an EP.

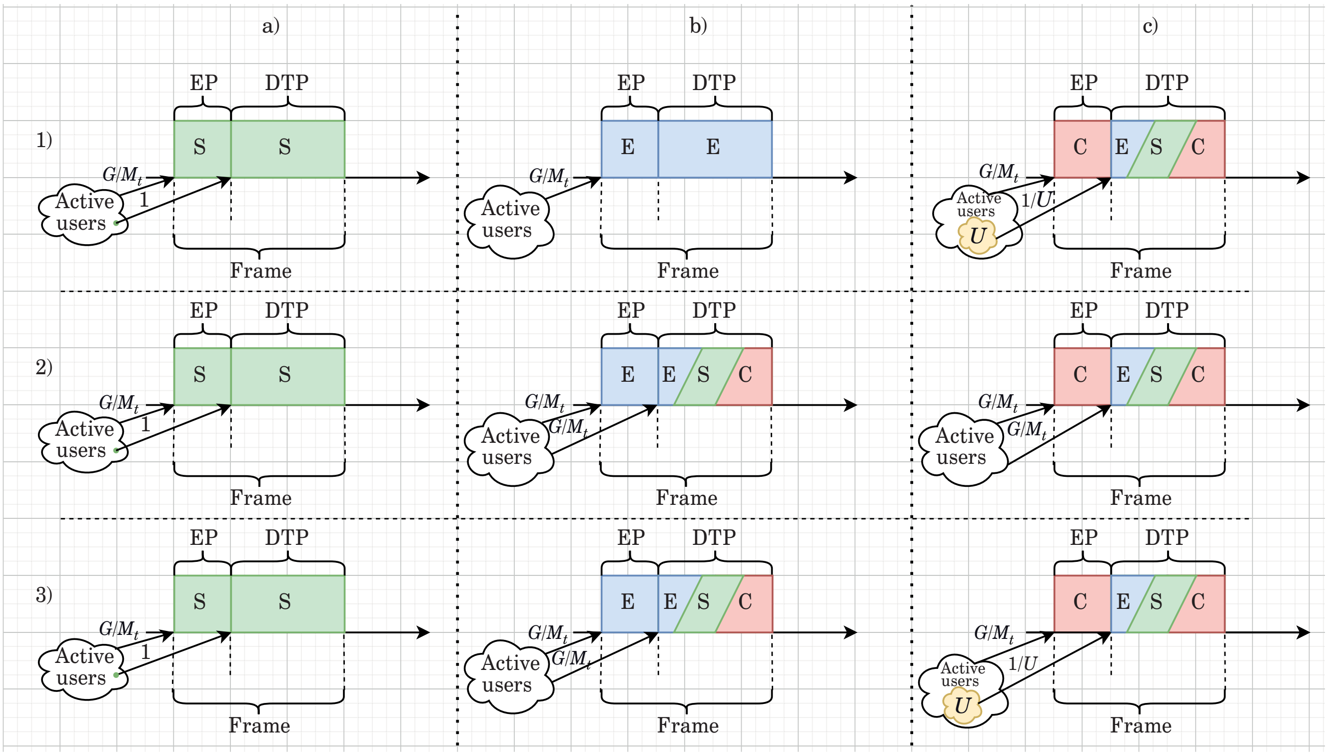
Steps 1 and 2 defined an EP and steps 3 and 4 define DTP.

Possible situations when using the 3 described algorithms are illustrated in Fig. 2: 1) refers to the ALOHA algorithm with a EP, 2) refers to the 2-step ALOHA algorithm, and 3) refers to the algorithm considered in this paper. In addition, a) means the "Success" event in the EP, b) means the "Empty" event in the EP, and c) means the "Conflict" event in the EP.

According to the possible events presented in Fig. 2, if the "Success" event occurs on the EP, the

■ ***Table 1.*** Algorithm of operation the system with ALOHA-based algorithms with early feedback

Number of step	ALOHA with EP	2-step ALOHA	Combined ALOHA
Step 1	With probability p_{EP} each user decides whether to transmit the preamble and with probability $1 - p_{EP}$ decides not to transmit p_{EP} can be calculated using the formula: $P_{EP} = \min\left(1,\ \frac{G}{M_t}\right)$ Users who decide to transmit randomly select and transmit one of the L preambles		
Step 2	The base station informs users about the number of detected preambles via the feedback channel	Through the feedback channel, the base station informs all users about the event that occurred in the channel (one of the following events is possible: "Success" — 1 preamble received, "Empty" — 0 preambles received, "Conflict" — more than 1 preamble received)	The base station informs users about the number of detected preambles via the feedback channel
Step 3 (if Success on EP)	The user transmits during DTP with probability $p_{DTP} = 1$ for 1 detected preamble in the channel		
Step 3 (if Empty on EP)	Nobody transmit on DTP	The user will attempt to transmit data during the DTP phase with the following probability: $p_{DTP} = \min\left\{1,\ \frac{G}{M_t}\right\}$	All users with a message ready to transmit during DTP will transmit with probability: $p_{DTP} = \min\left(1,\ \frac{G}{M_t}\right)$
Step 3 (if Conflict on EP)	The user transmits during DTP with a probability of following probability: $p_{DTP} = \frac{1}{U}$	The user will attempt to transmit data during the DTP phase with the following probability: $p_{DTP} = \min\left\{1,\ \frac{G}{M_t}\right\}$	The user transmits during DTP with a probability of following probability: $p_{DTP} = \frac{1}{U}$
Step 4	At the end of each frame, the base station notifies all users via the feedback channel about the event that occurred on the channel. Users exit the system only if the data transmission is successful; otherwise, they return to step 1 of the algorithm		



■ ***Fig. 2.*** Frame diagram of RA algorithms for various events after EP

"Success" event will occur for all algorithms on the DTP. According to column "b" of Fig. 2, in the case of the "Empty" event on the EP for the ALOHA algorithm with the EP on the DTP, the "Empty" event will occur. With the same event on the EP for the other two algorithms on the DTP, any event can occur ("Empty", "Success", "Conflict"). If the "Conflict" event occurs on the EP, one of the following events may occur on the DTP in all algorithms: "Empty", "Success", "Conflict". For each of these events, an arrow from the entire set of users, from a specific user, or from a subset of users indicates at which phase the decision to transmit is made, and the inscription above the arrow indicates the probability of making such a decision.

To further analyze the combined ALOHA algorithm, we describe the system of assumptions.

System model

In [14], a system model is introduced that is defined by five assumptions. This model assumes the following: the entire system operation time is divided into frames of equal length, consisting of a exploration phase and data transmission phase. The number of active users at the beginning of each frame is known. The system operates with a fixed number of unique preambles. The time required to transmit each preamble is uniform. It is important to note that information about the number of received preambles is transmitted without errors via the feedback channel. The base station reliably determines the number of received preambles. It is assumed that the number of users entering the system during a frame is a random variable distributed according to the Poisson law with parameter λ. Each user has one message ready to be transmitted and after successful transmission leaves the system. Data transmitted by multiple users during DTP cannot be reliably detected at the base station. Each phase ends with one of three events: "Empty", "Success", "Conflict".

This model is fully characterized by two parameters and the users operation algorithm: L is the number of preambles, λ is the input arrival rate.

This model is based on the model from [12], but has some differences. The differences are presented in Table 2.

Throughput with knowledge of the exact number of active users

For further comparison of the algorithm under consideration in this paper with the algorithms presented in [14] and [15], we obtain a formula for calculating the throughput with exact knowledge of the number of active users. It is important to note that similar formulas for the ALOHA algorithms

■ ***Table 2.*** *Differences in model assumption systems*

Number of difference	System from this work	System discussed in [12]
1	Number of unique preamble is finite	Unlimited number of unique preambles
2	System has one channel	System has large number of independent channels
3	Users transmit the preamble with some probability different from one	Users always transmit the preamble with probability one
4	Users are exits the system only if the transfer is successful. In this case, the system can have retransmissions	Users are exits the system after the first data transmission

with exploration phase and the 2-step ALOHA algorithm were obtained in the papers [14] and [15] regarding.

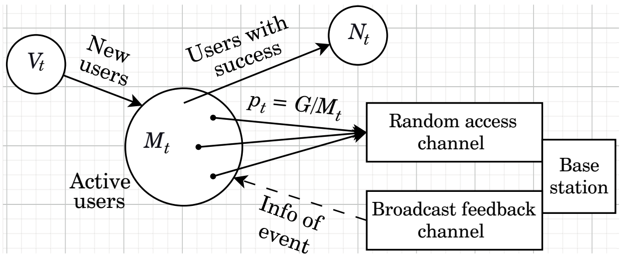
Figure 3 shows the system operation diagram for the case when all users know the total number of active users. In each frame with number t, new users enter the system, the number of which is equal to V_t. Active users, the number of which is equal to M_t, using information about the number of active users, independently of each other in the EP phase make a decision on transmission with probability $p_t = G/M_t$. The decision to transfer to DTP is then made in accordance with the algorithm. Users who successfully transmit during the DTP phase leave the system. The number of such users is N_t.

The process of changing the number of active users is described by the following equality:

$$M_{t+1} = M_t - N_t + V_t.$$

It follows from this expression that the sequence $\{M_1, \ldots, M_t\}$ is homogeneous irreducible aperiodic Markov chain. Based on the results from [14], we will understand the throughput as the upper bound of the input arrival rate, up to which the Markov chain is ergodic:

$$T(G,\, L) = \sup_{\lambda}\{\lambda : Markov\, chain\, ergodic\}.$$



■ ***Fig. 3.*** Scheme of the system operation with a known number of active users

Using approaches from the works [14], it can be shown that the value of $T(G, L)$ can be calculated using the following equation:

$$T(G,\, L) = \lim_{n\to\infty} \Pr\{B_t \mid M_t = n\}. \qquad (1)$$

Let us show how to calculate the probability $\Pr\{B_t|M_t = n\}$. Based on the definition given earlier, this probability can be calculated as follows:

$$\Pr\{B_t \mid M_t = n\} = np_{EP}\left(1 - p_{Ep}\right)^{n-1} + \\ + \sum_{i=2}^{n} C_n^i p_{EP}^i \left(1 - p_{EP}\right)^{n-1} \Pr\{B_t \mid i\} + \\ + \left(1 - p_{EP}\right)^n np_{DTP}\left(1 - p_{DTP}\right)^{n-1},$$

where $\Pr\{B_t| \ i\}$ — is the probability of the event “Success” for i users and L preambles.

Probability $\Pr\{B_t| \ i\}$ was calculated in [14]. In this case, substituting the values $p_{EP} = p_{DTP} = {G}/{M_t}$, formula (1) will have the following form:

$$T(G, L) = \lim_{n\to\infty} \Pr\{B_t \mid M_t = n\} = Ge^{-G} + \\ + \sum_{i=2}^{\infty} \frac{G^i}{i!} e^{-G} \sum_{j=2}^{\min(i,L)} C_L^{L-j} \sum_{v=0}^{j} (-1)^v \times \\ \times C_j^v \left(1 - \frac{L - j + v}{L}\right)^i i \frac{1}{j}\left(1 - \frac{1}{j}\right)^{i-1} + Ge^{-2G}.$$

From the above formula it follows that for a given fixed L, the value of G affects the throughput. The function under consideration is monotone and unimodal for all $G \in (0; \infty)$. Thus, the throughput can be maximized for a given number of preambles L by the parameter G. Let us introduce the following notations: $T_{\max}(L) = \max_G T(G,\, L)$ and also $G_{\text{opt}}(L) = \arg\max_G T(G,\, L)$. The results of calculating these values for different numbers of preambles will be presented below in Table 6.

Throughput using the procedure for estimating the number of active users

In real systems, there is no way to reliably know the number of active users at the beginning of each frame to determine the probability of transmission during EP. To get closer to real conditions, one can use some procedure for estimating the number of active users in a frame, based on the approach proposed in [18]. In this case, assumption 6 is excluded from the system of assumptions.

Figure 4 shows the system operation diagram for the case when the number of active users is unknown. In each frame with number t, new users enter the system, the number of which is equal to V_t. The base station calculates the value of S_t according to a certain rule and transmits it via the feedback channel. Active users, the number of which is equal to M_t, using S_t, independently of each other in the EP phase make a decision on transmission with probability $p_t = G/S_t$. The decision to transfer to DTP is then made in accordance with the algorithm. Users who successfully transmit during the DTP phase leave the system. The number of such users is N_t.

The process of changing the estimate of the number of active users is described by the following equality (where $S_1 = 1$):

$$S_{t+1} = \max\left[1,\ S_t + aI\{A_t\} + bI\{B_t\} + cI\{C_t\}\right],$$

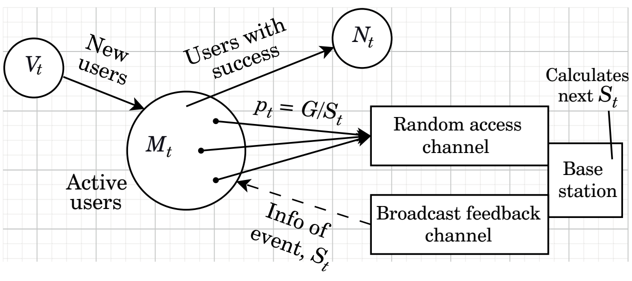
where $I\{*\}$ — is the indicator function. The coefficients a, b and c are defined as follows:

$$a = Q-1;\ b = Q-1;\ c = \frac{2}{(e-2)} + Q,$$

where Q — is an estimation parameter that depends on the number of unique preambles.

Next, the probability of p_{EP} is calculated using the following formula:

$$P_{EP} = \min\left(1,\ \frac{G}{S_t}\right).$$



■ ***Fig. 4.*** Scheme of the system operation with an estimated number of active users

Based on the above, the sequence of pairs (M_t, S_t) is a two-dimensional Markov chain.

Using the results of work [18], to determine the ergodicity conditions of a given Markov chain, we calculate the average drift for each component of a given Markov chain:

$$E[M_{t+1} - M_t \mid S_t = s, M_t = n] = \lambda - \alpha_i;$$
$$E[S_{t+1} - S_t \mid S_t = s, M_t = n] = = c + (a-c)\beta_i + (b-c)\alpha_i,$$

where α_i — conditional probability of the event "Success" on the DTP with n active users and an estimate s of the number of active users in the frame and β_i — conditional probability of the event "Empty" on the DTP with n active users and an estimate s of the number of active users in the frame.

Remark: The approach to estimating the number of active users for the ALOHA algorithm with a EP (algorithm 1) was considered in [14]. Also in [14] the coefficients a, b and c were proposed, and the optimal coefficient Q was calculated for different numbers of preambles. This paper examines in detail the application of the approach to estimating the number of active users for the 2-step ALOHA algorithm (algorithm 2), the combined ALOHA algorithm (algorithm 3), and provides the main results from [14] for algorithm 1 for comparison.

Table 3 presents the final formula for calculating the coefficients α_i and β_i for algorithm 1 and provides the derivation of formulas for calculating these coefficients for algorithms 2 and 3. In this case, the fact is taken into account that the number of users who decided to transmit to the EP is distributed according to the binomial law with parameters p_{EP} and n.

Following the work [18], to determine the ergodicity conditions, we introduce the following functions $\mu_n(\lambda, k)$ and $\mu_s(k)$:

$$\mu_n(\lambda, k) = \lambda - \lim_{\substack{n\to\infty \\ s\to\infty \\ k=n/s}} \alpha_i;$$

$$\mu_s(k) = c + (a-c)\lim_{\substack{n\to\infty \\ s\to\infty \\ k=n/s}} \beta_i + (b-c)\lim_{\substack{n\to\infty \\ s\to\infty \\ k=n/s}} \alpha_i. \quad (2)$$

Taking into account that during the limit transition $n \to \infty$, $s \to \infty$ and maintaining a constant relationship $k = n/s$ the binomial distribution transforms into a Poisson distribution with parameter $k = n/s$, and the limits defined above can be calculated using the formulas presented in Table 4.

It follows from [18] that in order to determine whether a Markov chain (M_t, S_t) is ergodic for some value λ, it is necessary to perform the following steps:

■ ***Table 3.*** *Formulas for calculating the coefficients* α_i *and* β_i

α_i	$i=1$	$\frac{nG}{s}\left(1-\frac{G}{s}\right)^{n-1}+\sum_{i=2}^{n}C_n^i\left(\frac{G}{s}\right)^i\left(1-\frac{G}{s}\right)^{n-i}\sum_{j=2}^{\min(i,L)}C_L^{L-j}\sum_{v=0}^{j}(-1)^v C_j^v\left(1-\frac{L-j+v}{L}\right)^i\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}$
	$i=2$	$np_{EP}\left(1-p_{EP}\right)^{n-1}+np_{DTP}\left(1-p_{DTP}\right)^{n-1}\left(\sum_{i=2}^{n}C_n^i p_{EP}^i\left(1-p_{EP}\right)^{n-i}\left(1-\left(\frac{1}{L}\right)^i\right)+\left(1-p_{EP}\right)^n\right)=$ $=\frac{nG}{s}\left(1-\frac{G}{s}\right)^{n-1}+\frac{nG}{s}\left(1-\frac{G}{s}\right)^{n-1}\left(\sum_{i=2}^{n}C_n^i\left(\frac{G}{s}\right)^i\left(1-\frac{G}{s}\right)^{n-i}\left(1-\left(\frac{1}{L}\right)^i\right)+\left(1-\frac{G}{s}\right)^n\right)$
	$i=3$	$np_{EP}\left(1-p_{EP}\right)^{n-1}+\sum_{i=2}^{n}C_n^i p_{EP}^i\left(1-p_{EP}\right)^{n-i}\Pr\{B_t\mid i\}+\left(1-p_{EP}\right)^n np_{DTP}\left(1-p_{DTP}\right)^{n-1}=$ $=\frac{nG}{s}\left(1-\frac{G}{s}\right)^{n-1}+\sum_{i=2}^{n}C_n^i\left(\frac{G}{s}\right)^i\left(1-\frac{G}{s}\right)^{n-i}\sum_{j=2}^{\min(i,L)}C_L^{L-j}\sum_{v=0}^{j}(-1)^v C_j^v\left(1-\frac{L-j+v}{L}\right)^i\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}+\left(1-\frac{G}{s}\right)^n\frac{nG}{s}\left(1-\frac{G}{s}\right)^{n-1}$
β_i	$i=1$	$\left(1-\frac{G}{s}\right)^n+\sum_{i=2}^{n}C_n^i\left(\frac{G}{s}\right)^i\left(1-\frac{G}{s}\right)^{n-i}\sum_{j=2}^{\min(i,L)}C_L^{L-j}\sum_{v=0}^{j}(-1)^v C_j^v\left(1-\frac{L-j+v}{L}\right)^i\left(1-\frac{1}{j}\right)^i$
	$i=2$	$\left(\sum_{i=2}^{n}C_n^i p_{EP}^i\left(1-p_{EP}\right)^{n-i}\left(1-\left(\frac{1}{L}\right)^i\right)+\left(1-p_{EP}\right)^n\right)\left(1-p_{DTP}\right)^n=\left(\sum_{i=2}^{n}C_n^i\left(\frac{G}{s}\right)^i\left(1-\frac{G}{s}\right)^{n-i}\left(1-\left(\frac{1}{L}\right)^i\right)+\left(1-\frac{G}{s}\right)^n\right)\left(1-\frac{G}{s}\right)^n$
	$i=3$	$\sum_{i=2}^{n}C_n^i p_{EP}^i\left(1-p_{EP}\right)^{n-i}\Pr\{A_t\mid i\}+\left(1-p_{EP}\right)^n\left(1-p_{DTP}\right)^n=$ $=\sum_{i=2}^{n}C_n^i\left(\frac{G}{s}\right)^i\left(1-\frac{G}{s}\right)^{n-i}\sum_{j=2}^{\min(i,L)}C_L^{L-j}\sum_{v=0}^{j}(-1)^v C_j^v\left(1-\frac{L-j+v}{L}\right)^i\left(1-\frac{1}{j}\right)^i+\left(1-\frac{G}{s}\right)^n\left(1-\frac{G}{s}\right)^n$

■ ***Table 4.*** *Formulas for calculating limits of the coefficients* α_i *and* β_i

$\lim\limits_{\substack{n\to\infty\\ s\to\infty\\ k=n/s}}\alpha_i$	$i=1$	$e^{-Gk}\left(Gk+\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}\Pr\{j\mid i\}\right)$
	$i=2$	$Gke^{-Gk}\left(1+e^{-Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\left(1-\left(\frac{1}{L}\right)^i\right)+1\right)\right)$
	$i=3$	$e^{-Gk}\left(Gk+\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}\Pr\{j\mid i\}+Gke^{-Gk}\right)$
$\lim\limits_{\substack{n\to\infty\\ s\to\infty\\ k=n/s}}\beta_i$	$i=1$	$e^{-Gk}\left(1+\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\left(1-\frac{1}{j}\right)^i\Pr\{j\mid i\}\right)$
	$i=2$	$e^{-2Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\left(1-\left(\frac{1}{L}\right)^i\right)+1\right)$
	$i=3$	$e^{-Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\left(1-\frac{1}{j}\right)^i\Pr\{j\mid i\}+e^{-Gk}\right)$

1) for a fixed value of λ, solve the equation $\mu_n(\lambda, k) = k\mu_s(k)$ for the unknown k;
2) check that the following condition is satisfied for all roots of the equation:

$$\mu_n(\lambda, k_i) < 0,$$
$$\mu_s(k_i) < 0.$$

3) if the condition is satisfied, then the Markov chain for a given value of λ is ergodic.

Taking into account formula (2) and the formula from Table 4, the final formulas for calculating the functions $\mu_n(\lambda, k)$ and $\mu_s(k)$ are presented in Table 5.

Let the number of unique preambles L be given and the throughput value for the exact number of active users be known $T_{\max}(L)$. Then one can numerically find Q for which the two-dimensional Markov chain is ergodic at $\lambda = T_{\max}(L)$. The value of the estimation parameter Q at which the throughput value equal to $T_{\max}(L)$ is achieved will be denot-

■ ***Table 5.*** *Formulas for calculating coefficients* $\mu_n(\lambda, k)$ and $\mu_s(k)$

$\mu_n(\lambda, k)$	$i = 1$	$\lambda - e^{-Gk}\left(Gk + \sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}\Pr\{j \mid i\}\right)$
	$i = 2$	$\lambda - Gke^{-Gk}\left(1 + e^{-Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\left(1-\left(\frac{1}{L}\right)^i\right)+1\right)\right)$
	$i = 3$	$\lambda - e^{-Gk}\left(Gk + \sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}\Pr\{j \mid i\} + Gke^{-Gk}\right)$
$\mu_s(k)$	$i = 1$	$c + (a-c)e^{-Gk}\left(1 + \sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\left(1-\frac{1}{j}\right)^{i}\Pr\{j \mid i\}\right) + (b-c)e^{-Gk}\left(Gk + \sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}\Pr\{j \mid i\}\right)$
	$i = 2$	$c + (a-c)e^{-2Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\left(1-\left(\frac{1}{L}\right)^i\right)+1\right) + (b-c)Gke^{-Gk}\left(1 + e^{-Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\left(1-\left(\frac{1}{L}\right)^i\right)+1\right)\right)$
	$i = 3$	$c + (a-c)e^{-Gk}\left(\sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\left(1-\frac{1}{j}\right)^{i}\Pr\{j \mid i\} + e^{-Gk}\right) + {}$ $+ (b-c)e^{-Gk}\left(Gk + \sum_{i=2}^{\infty}\frac{(Gk)^i}{i!}\sum_{j=2}^{\min(i,L)}\frac{i}{j}\left(1-\frac{1}{j}\right)^{i-1}\Pr\{j \mid i\} + Gke^{-Gk}\right)$

■ ***Table 6.*** *Maximum throughput and the corresponding parameter values G and Q for combination algorithm of ALOHA*

L	$T_{\max}(L)$	$G_{\text{opt}}(L)$	$Q_{\text{opt}}(L)$
1	0.522	0.757	0.020
2	0.572	0.900	0.103
4	0.601	1.000	0.188
8	0.617	1.050	0.249
16	0.625	1.080	0.286
32	0.628	1.100	0.305
64	0.630	1.100	0.316
128	0.631	1.100	0.321

■ ***Table 7.*** *Maximum throughput and the corresponding parameter values G and Q for 2-step algorithm of ALOHA*

L	$T_{\max}(L)$	$G_{\text{opt}}(L)$	$Q_{\text{opt}}(L)$
1	0.521	0.76	0.020
2	0.562	0.90	0.247
4	0.582	0.96	0.329
8	0.591	0.98	0.354
16	0.596	0.99	0.362
32	0.598	1.00	0.366
64	0.599	1.00	0.367
128	0.599	1.00	0.367

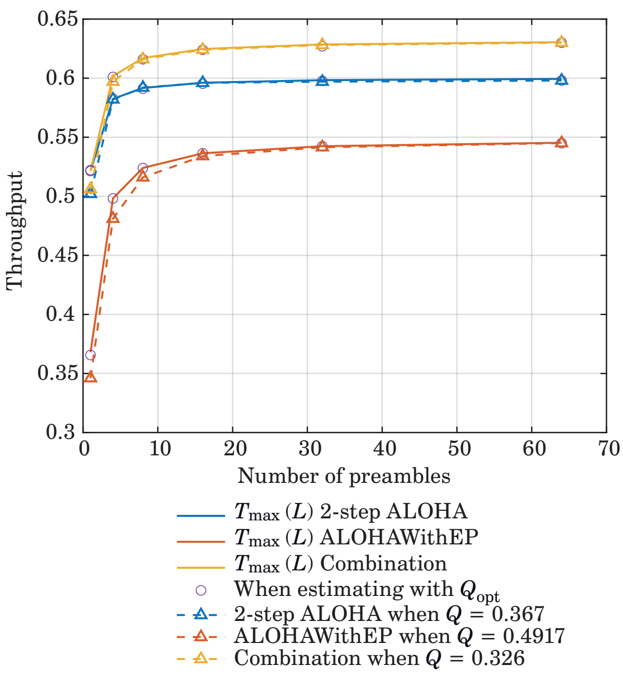
ed as $Q_{opt}(L)$. The values for the combined ALOHA approach are presented in Table 6. Similar values for the 2-step ALOHA algorithm are presented in Table 7.

Impact of the estimation procedure parameter on throughput

The dependences of the throughput on the number of preambles L for the considered algorithms are presented in Fig. 5. The solid lines show the dependence of the throughput on the number of preambles L for a known number of active users. The "circle" marker shows the throughput values for different values of the number of unique preambles L, an unknown number of active users and the use of $Q_{opt}(L)$ in the estimation procedure. The dotted lines with the "triangle" marker mark the corresponding values of the throughput when using a fixed value of Q in the estimation procedure for different numbers of preambles L. This value of Q for each of the algorithms under consideration is chosen as the value of $Q_{opt}(L)$ when the number of preambles L tends to infinity.

The following conclusions can be drawn from the presented dependencies.

1. With an unknown number of active users and using the $Q_{opt}(L)$ value in the estimation procedure, the same values of maximum throughput can be achieved as with exact knowledge of the number of active users.



■ ***Fig. 5.*** Throughput versus number of unique preambles

■ ***Table 8.*** *Values of T(L) with non-optimal value Q*

L	$T_1(L)$	$T_2(L)$	$T_3(L)$
1	0.346	0.502	0.506
2	0.423	0.555	0.564
4	0.481	0.582	0.597
8	0.516	0.592	0.616
16	0.534	0.596	0.624
32	0.542	0.597	0.628
64	0.545	0.598	0.63
128	0.547	0.599	0.631

2. When the number of active users is unknown and a fixed value of Q is used in the estimation procedure, the maximum throughput values are lower than $T_{max}(L)$. However, this difference rapidly decreases as the number of preambles increases. Next, we will show how to quantitatively evaluate the value of the relative gain.

The quantitative value of the relative loss can be calculated using the following formula:

$$\Delta_i = \frac{T_{max,i}(L) - T_i(L)}{T_{max,i}(L)},$$

where i — index of the algorithm; $T_{max,i}(L)$ — maximum throughput for the algorithm with index i, the values of $T_i(L)$ for each of the algorithms are presented in Table 8.

The relative loss when using a non-optimal Q (in our work, we take Q for an infinite number of preambles) does not exceed 6% for any number of preambles, and starting with $L = 30$, the loss is no more than 0.2%.

Note that for algorithm 1 the values in Table 8 were obtained with $Q = 0.491$, for algorithm 2 the values in Table 8 were obtained with $Q = 0.367$ and for algorithm 3 the values in Table 8 were obtained with $Q = 0.326$.

Conclusion

The paper presents a comparative analysis of random multiple access algorithms with early feedback based on ALOHA. A model of a system is considered, built on the basis of a model of a system with random multiple access and early feedback, proposed in [12]. Unlike work [12], the system has only one channel, a limited number of preambles, and to avoid message losses due to conflicts in the multiple access channel, retransmissions are carried out in accordance with a certain algorithm. We briefly describe two previously considered algorithms based on ALOHA with early feedback from [14] and [15] and introduce a new algorithm, which is a combination of the works [14, 15]. The operation of these algorithms is described from a unified point of view. For all three algorithms presented in this paper, two variants are considered: for the first variant, the number of active users is considered known, for the second variant, the number of active users is unknown and some estimation procedure is used. For each of the three algorithms, a procedure for estimating the number of active users is described, which is specified by one parameter Q. For some values of the number of preambles, the optimal parameter Q is determined to maximize the system throughput. For each algorithm, the dependence of the throughput on the number of unique preambles is investigated for the first and second variants. It is shown that when using the optimal value of Q, the throughput of the second variant is equal to the throughput of the first variant, that is, the considered estimation procedure allows, in the absence of information on the number of active users, to obtain the same throughput as in the presence of such information. It is also shown that when using a fixed value of the parameter Q for any number of preambles, equal to the optimal Q for an infinite number of preambles, the throughput loss is no more than 6% and decreases rapidly with increasing number of preambles. Starting from $L = 30$ the loss does not exceed 0.2%.

The obtained results show that the combined algorithm proposed for the first time in this paper, obtained on the basis of algorithms from works [14] and [15], allows obtaining the highest value of throughput for any number of unique preambles. In addition, this modification of the algorithm allows exceeding the maximum value of the throughput $e^{-1}(2 - e^{-1}) \approx 0.6004$ for systems with early feedback, first presented in [12] for a system without retransmissions and an unlimited number of unique preambles.

The considered combined algorithm based on ALOHA with early feedback can be used in the RA channel of future generations of wireless networks.

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Сравнительный анализ алгоритмов случайного множественного доступа с ранней обратной связью, построенных на базе АЛОХА

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Введение: одним из вариантов повышения пропускной способности системы случайного множественного доступа является использование ранней обратной связи, под которой понимается быстрый ответ базовой станции после приема преамбул. В работе ранняя обратная связь рассматривается для систем множественного доступа, использующих разные алгоритмы на базе АЛОХА. **Цель:** провести сравнительный анализ зависимости максимальной пропускной способности от числа уникальных преамбул алгоритмов случайного доступа на основе алгоритма АЛОХА с ранней обратной связью. **Результаты:** рассмотрены алгоритм АЛОХА с фазой исследования, 2-шаговый алгоритм АЛОХА и комбинация двух этих алгоритмов. Проведен сравнительный анализ алгоритма АЛОХА с фазой исследования, 2-шагового алгоритма АЛОХА и комбинированного алгоритма для варианта с известным числом уникальных абонентов и варианта с оценкой числа активных абонентов. Показано, что использование процедуры оценки числа активных абонентов позволяет достичь аналогичных значений зависимости максимальной пропускной способности от числа уникальных преамбул, что и для первого варианта с известным числом активных абонентов. Также демонстрируется, что использование фиксированного параметра, влияющего на процедуру оценки, равного оптимальному значению этого параметра для бесконечного числа преамбул, имеет проигрыш, не превышающий 6 % для любого числа преамбул и 0,2 % при 30 и более преамбулах. **Практическая значимость:** предложен новый алгоритм на базе АЛОХА с ранней обратной связью, позволяющий повысить максимальную пропускную способность системы по сравнению с ранее известными алгоритмами этого класса. Данный алгоритм может быть использован в канале случайного доступа сетей будущих поколений. **Обсуждение:** в рамках проведенного анализа не учитывалось влияние числа каналов, используемых в системе, что может быть дальнейшим направлением исследования.

Ключевые слова — АЛОХА, случайный доступ без выделения грантов, фаза исследования на основе преамбул, оценка, пропускная способность, эргодичность, марковская цепь.

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