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HEURISTICS OF CHANNEL ALLOCATION IN RADIO NETWORKS

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Introduction: *Any radio network (cell) providing service to a geographical region is associated with certain interference environment described by so-called compatibility matrix which, in turn, defines the required frequency limitations between separate cells. An engineering approach to fixed channel allocation (frequency planning) could be described as a trial to find such a frequency plan which would satisfy all the matrix constraints and would have the shortest width (span). The combinatorial nature of this problem makes it unrealistic to obtain the optimal solution. The only way to solve it is to use a certain set of heuristic algorithms based on the features of the compatibility matrix.* ***Purpose:*** *Our goal is to obtain statistically stable conclusions about relative efficiency of euristic algorithms (known and proposed ones) tested on benchmark problems (various matrices).* ***Methods:*** *The set of algorithms being compared includes both deterministic and stochastic ones. They all implement sequential trials to assign frequencies to networks, according to a certain ordering of the latter. The implemented "expert" system generates the required number of compatibility matrices with various characteristics. Each matrix represents a specific frequency allocation problem which is to be solved using the set of algorithms. Some algorithms are based on a simple ordering of networks during the process of frequency planning, while others also include ordering of frequencies themselves. As the tools for achieving an "almost best" frequency plan, i.e. the lower bound of its span, two adaptive random search algorithms were proposed and tested.* ***Results:*** *Algorithms with dual ordering (of networks and frequencies) show significant gain in effectiveness. The spans obtained with their help usually are shorter at least by some percents (sometimes by 10%) than those obtained by algorithms with simple ordering of only networks. The proposed adaptive random search algorithms provide that the frequency planning system is estimated to be close to "almost optimal".* ***Practical relevance:*** *Real frequency assignment problems should be solved by a set of heuristic algorithms with subsequent choice of the best result. The proposed algorithms with dual ordering have undoubted preference for including into the set used by Spectrum Manager.*

Keywords *— Frequency Planning, Adjacent Constraints, Sequential Scheduling Algorithms, Adaptive Random Search.*

Introduction

The increasing demands on the radio spectrum due to developing communication needs are outpacing the expansion of the available frequency bands. The Spectrum Manager (SM), trying to solve the problem of the efficient use of the frequency resource allocated for the service, must take into account certain constraints providing an interference-free environment for each of the applicants for assignment. In radio networks, for transmitters located at different sites, the frequency plan to be sought must answer, first of all, to the set of frequency-distance separation conditions, or so-called adjacent constraints:

$$\left|f_{ip}-f_{jq}\right|\geq m_{ij}\Delta, \qquad (1)$$

where the double index designates the allocation of the channel with frequency f_i to the p-th applicant, f_j to the q-th one correspondingly. If, as usual, the available spectrum represents a frequency grid with equal step Δ, m_{pq} is integer. The adjacent constraints (1) depend on the radio propagation, the required Signal-to Interference ratio at the most distant point of the service zone and mutual space distance d_{pq} between these applicants. If d_{pq} increases, then m_{ij} decreases and vice versa.

It is worth to separate the frequency planning to two related to each other problems. In the framework of the first of them SM tries to minimize the number of applicants which did not succeed to obtain a free-interference assignment with the help of frequency band with the given length F. The purpose of the second problem is minimization of F itself on condition that all applicants obtain such an assignment.

In cellular networks with regular (usually hexagonal) structure optimal frequency assignment, based on the principle of periodic frequency reuse, is achieved elementary. However, in practice, even in cellular communication networks and, naturally, in other civil and military applications the space regularity is not more than a kind of idealization [1].

In the real space distribution of applicants (transmitters) optimal solution is computationally unrealistic even for a very moderate size of the set of applicants, not exceeding some dozens. The near-optimal assignment for a priory known set of applicants may be achieved with the help of algorithms with a greedy heuristic [2–4].

Constraint and Availability Matrices

Any process of frequency allocation represents certain sequence of successive steps of finding pairs of an applicant and an available frequency. For the formal description of the problem let define the *adjacent* **A** and *availability* **B** matrices which are updated after each step of the assignment process. At the moment of the first step **A**(0) has fixed $R \times R$ dimension where R is the number of applicants with a priori known frequency-distance separation conditions. The matrix is symmetrical one with zeroes in the main diagonal and with other elements (integers from 0 to m_{pq}) obtained from (1). By deleting of one row and one column after n-th succeeding assignment the dimension of **A**(n) is reduced to $(R-n) \times (R-n)$, where $R-n$ is the number of applicants currently deserving assignment.

We suppose that there are M equally separated (ordered from 1 to M) frequency channels allocated for the service. At any step of the assignment process we may consider the *availability* binary matrix **B**(n) with dimension $(R-n) \times M$. The element $b_{(R-n),i}$ if the frequency f_i can not be assigned to the $(R-n)$ — the applicant and it is equal to zero in vice versa. Matrix **B**(n) is being updated following (1) as a result of every succeeded assignment performed.

Finite Allocation Strategies

In the most of theoretical publications devoted to the frequency planning problem, the considered network is interpreted as an non oriented graph with weighted edges [5–7], and the assignment process itself is interpreted as the regular coloring of this graph vertices, which means rigorous satisfaction of adjacent constraints. In the framework of this model the problems consists in more or less exact estimation of the chromatic number of the corresponding graph. As well known, it is an *NP*-hard, and so there the is no algorithm of its solution except the exhaustive search even for a non weighted graph. The more it is true for a weighted case. The second problem which SM has sometimes to solve (minimization of the number of applicants which did not get assignment) now can be interpreted as maximization of the number of graph vertices which may be «*F* colored».

All algorithms of the graph vertices coloring are realized as the sequential combined considerations of the current list of applicants and of the list of frequencies allocated for the service. Namely this act results in assignment of the frequency f_{ip} for the p-th applicant, where $1 \le i \le M$, $f_{i+1} - f_i = \Delta$, $1 \le p < P$. The obtained span is, evidently, defined as $F = (i_{\max} - 1)\Delta$.

Any heuristic at each step of the corresponding algorithm manipulates with **A**(n) and **B**(n) and may be defined as a certain operator F[**A**(n), **B**(n)]. The most evident approach is based on consideration of sums of the elements in the rows of those matrices

$$sum_{\mathbf{A}(n),p} = \sum a_p, \; sum_{\mathbf{B}(n),p} = \sum b_p, \qquad (2)$$

which include only the applicants which need assignment after n successful steps. The greedy algorithms have recursive nature and operates with permanently updated values (2). The most popular group of them uses certain ordering of applicants, and the optional frequencies are considered according their increasing. As an example, the algorithm known as «the biggest first» (*BF*), where the next applicant to be serviced is that with maximal first sum in (2), may be considered. After each act of assignment matrix **A** is reduced and all sums $sum_{\mathbf{A}(n),i} = \sum a_p$ for the applicants until not serviced are recalculated. An alternative to this algorithm is «the lowest last» (*LL*) heuristic. It is based on the inverted ordering of applicants.

There are two modifications of those algorithms which might be defined as «applicants before frequency» (*BFAF*) which is based only on the list of ordered applicants. The second modification includes reordering of applicants and reordering of frequencies, i.e. the frequency with lowest value of $sum_{\mathbf{B}(n),i} = \sum_{p=1}^{R-n} b_{pi}$ is being «loaded» by applicants from the list and only after its full saturation the next frequency with minimum value of $sum_{\mathbf{B}(n+1),i}$ is being considered. This approach is used in some of the algorithms to be compared and all ones from this group has the added abbreviation *FA*.

The most of the known greedy heuristics share the common property: they are based on consideration of the so called assignment difficulty. Really, $sum_{\mathbf{A}(n),p}$ in (2) represents the sum of constraints related to the p-th applicant, and $sum_{\mathbf{B}(n),i}$ characterizes the level of the unavailability of the frequency unavailability. In other words, they realize heuristic principle «first difficult». However, the assignment difficulty itself is not more than a common sense definition.

Adaptive Random Search of Frequency Plan with Minimum Span

The optimal (or rather close to it) ordering of applicants leading to minimum span may be constructed with the help of certain adaptive stochastic search (*AS*) which does not need a priory definition of the assignment difficulty. Let the number of frequencies required for to all applicants in a random sequence of applicants $\mathbf{V}_0 = \{v_0^{(1)}, v_0^{(2)}, \ldots, v_0^{(p)}, \ldots, v_0^{(R)}\}$ is M_1. A subsequence $\mathbf{V}_1^{f_{M_1}}$ of applicants for which the maximal frequency f_{M_1} was assigned is formed. For all its elements random uniformly distributed penalties are assigned and they are put in the head of the new sequence $\mathbf{V}_1 = \{v_1^{(1)}, v_1^{(2)}, \ldots, v_1^{(p)}, \ldots, v_1^{(R)}\}$. At the initial step all applicants of

$\mathbf{V}_0 = \left\{v_0^{(1)}, v_0^{(2)}, \ldots, v_0^{(p)}, \ldots, v_0^{(R)}\right\}$ has zero «penalties». During the next step the algorithm tries to satisfy all R applicants with the help of $M_2 = M_1 - 1$ frequencies. If it occurs unsuccessful, penalties are assigned to the group of non-serviced applicants $\mathbf{V}_1^{f_{M12}}$ and it is shifted to the head of the new sequence $\mathbf{V}_2 = \left\{v_2^{(1)}, v_2^{(2)}, \ldots, v_2^{(R)}\right\}$, ordering by the obtained penalties. The cycle is repeated till the successful attempt or till the last (fixed a priory) iteration. The evident advantage of this algorithm is the fact that it converges to the exact estimation of the minimal span.

The following modification of this algorithm may significantly increase the convergence rate. In addition to reordering of the applicants' list we propose also reordering of frequencies after every assignment of the current estimation of the span. Minimally loaded frequencies also get random penalties and form the tail of the updated frequencies list. This algorithm is called below «double tuning» (*DT*).

Comparison of the Strategies

To compare the effectiveness of above considered algorithms, we used three kinds of test problems in our data set. The first is given by networks with only co-channel constraints (networks represented by ordinary graphs with binary edges density d_1). The second group was presented by the networks with co-channel and first adjacent channel constraints (networks represented by a graph with edges equal to one or two with edges densities d_1 and d_2). The third group consisted of the networks with co-channel, first and second adjacent channel constraints (with densities d_1, d_2, d_3). Dimensions of the networks varied from 20 to 80. Matrices of constraints **A**(0) were generated randomly and their number was chosen to be sufficient for the stable averaging of the values of the span achieved with the help of different algorithms.

Results for the average span for two dimensions of networks (20 and 80) are presented in Tables 1 and 2 respectively. In addition to above mentioned

■ ***Table 1.*** The value of average span obtained by different algorithms for networks with 20 transmitters

Algorithm	Adjacent constraints density								
	d_1			d_1 d_2			d_1 d_2 d_3		
	0.25	0.50	0.75	0.50 0.16	0.33 0.33	0.16 0.50	0.50 0.25 0.125	0.125 0.25 0.375	0.125 0.25 0.25
BFAF	4.2	7.0	9.9	10.9	12.6	15.7	15.5	18.6	20.7
BFFA	4.2	6.9	9.8	10.6	12.5	15.4	15.4	18.2	20.6
LLAF	4.1	7.1	9.8	11.0	12.7	15.6	15.6	18.7	20.9
LLFA	4.0	6.8	9.7	10.5	12.5	15.3	15.2	18.0	20.2
TrAF	5.0	7.6	13.4	13.0	14.3	17.7	17.8	19.7	24.1
TrFA	4.8	7.2	13.2	12.2	14.7	15.3	17.1	19.1	22.2
AS	4.0	6.5	9.6	8.7	10.8	13.6	12.8	17.7	19.5
DT	4.0	6.5	9.5	8.5	10.7	13.4	12.5	17.3	19.1

■ ***Table 2.*** The value of average span obtained by different algorithms for networks with 80 transmitters

Algorithm	Adjacent constraints density								
	d_1			d_1 d_2			d_1 d_2 d_3		
	0.25	0.50	0.75	0.50 0.16	0.33 0.33	0.16 0.50	0.50 0.125 0.125	0.25 0.25 0.25	0.125 0.375 0.25
BFAF	9.6	17.2	26.8	28.2	32.8	38.8	35.8	47.6	55.2
BFFA	9.5	17.0	26.1	27.7	32.0	38.4	35.2	47.3	54.1
LLAF	9.4	17.6	26.6	27.6	32.2	38.4	35.6	47.4	55.3
LLFA	9.3	17.2	25.8	27.2	31.9	38.2	35.0	47.0	55.1
TrAF	11.2	19.2	30.0	30.1	36.2	42.4	39.0	53.0	58.3
TrFA	11.4	19.1	29.8	29.8	35.9	42.0	38.9	52.9	58.0
AS	9.1	14.2	23.8	27.0	30.2	36.8	34.0	44.2	52.1
DT	9.0	13.9	23.6	26.7	30.1	36.3	33.7	43.8	51.7

algorithms the trivial one designated as *Tr1* without applicants and frequencies ordering and *Tr2* with frequencies ordering were included in testing.

Conclusions

From consideration of the Tables content the following conclusions may be done.

1. The effectiveness of any greedy heuristic is better than of an algorithm without applicants ordering at least by 15 %.

2. The difference of algorithms effectiveness increases with the network dimension.

3. Maximal difference ineffectiveness for networks with only co-channel constraints is observed for the case of density 0.5.

4. Effectiveness of algorithms from the group FA is always higher than of algorithms from the group AF, so ordering of frequencies occurs to be positive.

5. The proposed algorithm of adaptive search asymptotically reduces to the exact value of minimal span. Its gain in comparison to the considered greedy algorithms varies from 5 to 15 %. Ordering of frequencies leads to the shortage of the required number of iterations, so the «double tuning» heuristic is more effective than «simple adaptive search».

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Об эвристиках, используемых при назначении частот в радиосетях

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Введение: любая радиосеть, обслуживающая некий географический район, ассоциируется с определенной помеховой обстановкой, которая описывается так называемой матрицей совместимости, которая в свою очередь определяет требуемые частотные ограничения между отдельными радиосетями. Инженерный подход к решению проблемы назначения частот может быть интерпретирован как попытка найти такой частотный план, который бы удовлетворял всем ограничениям матрицы совместимости и имел бы минимальную протяженность. Комбинаторная природа этой задачи делает получение оптимального решения нереальным, и единственной альтернативой оказывается использование некоего набора эвристических алгоритмов, основанных на свойствах указанной матрицы. **Цель:** статистически устойчивые выводы о сравнительной эффективности эвристических алгоритмов (известных и предлагаемых), которые тестируются путем рассмотрения различных матриц. **Методы:** группа сравниваемых алгоритмов включает в себя как детерминированные, так и стохастические. И те и другие реализуют последовательные попытки назначить частоты сетям в соответствии с определенной упорядоченностью последних. Реализованная «экспертная» система генерирует требуемое количество матриц совместимости. Каждая матрица представляет собой конкретную проблему частотного планирования, которая должна быть решена с помощью всех алгоритмов группы. Некоторые алгоритмы используют простое упорядочивание сетей в процессе решения, в то время как другие включают в себя также определенное упорядочивание частот. Получение нижней границы протяженности частотного плана, то есть «почти наверняка» лучшего возможного частотного плана, реализовалось с помощью двух предложенных модификаций адаптивного случайного поиска. **Результаты:** алгоритмы с двойным упорядочиванием (сетей и частот) демонстрируют существенный выигрыш в эффективности. Частотные планы, полученные с их помощью, оказываются на несколько процентов короче (иногда выигрыш достигает 10 %), чем планы, полученные с помощью алгоритмов с упорядочиванием только сетей. Предложенные алгоритмы адаптивного случайного поиска обеспечивают системе частотного планирования оценку ее близости к «почти оптимальной». **Практическая значимость:** реальные задачи частотного планирования для радиосетей должны решаться с помощью группы эвристических алгоритмов с последующим выбором лучшего результата. Предложенные алгоритмы с двойным упорядочиванием, несомненно, эффективнее и должны использоваться прежде других.

Ключевые слова — частотное планирование в радиосетях, ограничения по соседним каналам, последовательные алгоритмы оптимизации, адаптивный случайный поиск.