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MODEL OF DEEP FADING

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Introduction: *Fading, i. e. random changes in the level of a radio signal, is one of the main problems in signal processing, as the result strongly depends on the fading depth. The distribution of the received signal envelope is sometimes left-shifted relative to the Rayleigh law. A radio propagation channel with such a sub-Rayleigh fading may be considered a "critical" one.* ***Purpose:*** *Synthesizing a model of a narrow-band random process with an envelope distribution left-shifted relative to the Rayleigh law.* ***Results:*** *The synthesis of a model is based on representing the process as a reaction of a stable dynamic system to white Gaussian noise excitation. We have obtained nonlinear stochastic second-order differential equations to simulate the fading of a radio signal having an envelope with Nakagami or Weibull distribution. The envelope is considered a Markov continuous process. An analytical expression for the envelope correlation function is obtained. It is shown that, at least, for Nakagami fading, the correlation interval of the envelope almost does not depend on its depth.* ***Practical relevance:*** *In various applications, including indoor radio communication, the level of the received signal can be critically low during long time intervals. The proposed model used as a simulator core for such a propagation channel provides the opportunity to evaluate the performance quality of a communication system at the stage of its development.*

Keywords *— Sub-Rayleigh Fading, Markovian Diffusion Processes, Stochastic Differential Equations, Nakagami Distribution Envelope, Weibull Distribution Envelope, Radio Signal Propagation Model.*

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Introduction

A radio channel is a rather complex medium. It is not repeatable, so the problem of the radio propagation modeling is a part of the system design, and there is a challenge to assure the required system's quality during all its stages. Any channel model is based on generation of the so-called fading carrier, i.e. its reaction on the pure sinusoid excitation. The channel output, in turn, represents a narrow band random process with certain statistical characteristics of its envelope and phase. The dominant approach to Simulation of fadind carrier is based on the suppose that it represents a sum of diffuse and line-of-sight components. The diffuse stationary component results from a large number of independent and identically distributed scatters. It might be considered as a product of multiplying the in-phase and quadrature components of the transmitted carrier by two mutually non- correlated band-limited white Gaussian noises. If the both have the same dispersion and zero mean then the fading carrier envelope is Rayleigh distributed and does not depend on the carrier phase, which, in turn, has a uniform probability density function (PDF) [1].

If the line-of-sight component of the received signal is not equal to zero, the fading carrier becomes nonstationary, and the PDF of its envelope is right shifted with respect to the Rayleigh law. This, so called generalized Gaussian fading model usually continues to be valid, but this assertion is not true in the case of a fading, which is deeper than the Rayleigh one. Such phenomenon takes place if only a limited number of scatters actually contribute to the received signal. It is observed in the High-Frequency (HF) channel with significant variability [2], in Very-High-Frequency (VHF) indoor communication channel [3]. It describes the unwanted phenomenon which usually accompanies target detection by sea microwave radar [4]. In all these cases the measured envelope PDF exhibits large deviations from the Rayleigh distribution. The fading carrier quadrature components continue to be non-correlated but now due their non-Gaussian nature they become mutually dependent [1]. Disregarding of this fact leads to misinterpretation of experimental data and to the system performance decreasing as now the receivers designed under the Gaussian assumption are not optimal [5]. As well as channels with such so-called sub-Rayleigh fading have relative lack of reliability, namely they must be considered in the attempts of obtaining estimation of the system lower bound performance.

The Model of the Fading Carrier

Throughout the paper, we assume that the simulated propagation channel is not frequency-selective, and so in the case of a sinusoidal transmitted signal the received signal (fading carrier) may be written as

$$x(t)=r(t)\cos(\omega_0 t+\theta), \qquad (1)$$

where $x(t)$ is a stationary process and the PDF of its envelope $P_r(r)$ is described by [6] the Nakagami law

$$P_r(r)=\begin{cases}\dfrac{2}{\Gamma(m)}\left(\dfrac{m}{\sigma^2}\right)r^{2m-1}\exp\left(-\dfrac{mx^2}{\sigma^2}\right), & r\ge 0\\ 0, & r<0\end{cases}, \quad (2)$$

or by the Weibull law

$$P_r(r)=\begin{cases}\alpha\beta r^{\alpha-1}\exp\left(-\beta r^{\alpha}\right), & r\ge 0\\ 0, & r<0\end{cases}. \quad (3)$$

In (2) $\Gamma(\cdot)$ is the gamma function, $m\ge 0.5$ is the fading parameter (the shape factor) and $2\sigma^2=\langle r^2\rangle$. The case of $m=1$ corresponds to the Rayleigh PDF and if $0.5\le m<1$ the fading is deeper (a sub-Rayleigh one). If $m=0.5$ PDF (2) is the one-sided Gaussian. If in (3) parameter $\alpha<2$ the corresponding fading is deeper than Rayleigh. If $\alpha=2$ the PDF is Rayleigh. If $\alpha-1$ it becomes one-sided exponential. Consideration of some experimental results [5] shows that in indoor channel m and α appear to be very close to those limiting values.

It is evident fact that a strong connection exists between the PDF of $x(t)$ and $P_r(r)$ principally gives an opportunity to generate $x(t)$ as a product of a synthesized process $r(t)$ and a Gaussian band pass process, but we propose a different approach consisting of the direct generation of the fading carrier by a second-order dynamic system excited by a white Gaussian noise (WGN). It is clear that generally this system must be nonlinear.

If the generating stochastic differential equation (SDE) is a stochastic modification of the Duffing equation with the operator

$$\ddot{x}+\mu\dot{x}+\omega_0^2 f(x)x=\sqrt{\frac{N_0}{2}}\xi, \quad (4)$$

where $\mu>0$; $f(x)$ is a positively defined function; $\xi(t)$ is WGN with unit power spectral density; ω_0 is the natural frequency of the oscillator (5), the envelope PDF $P_r(r)$ may be the Rayleigh one (in the linear case) or over-Rayleigh, but never sub-Rayleigh [7], so it is not appropriate for our purpose. Instead (4) we consider the following generating SDE

$$\ddot{x}+f(r)\dot{x}+\omega_0^2 x=\sqrt{\frac{N_0}{2}}\xi, \quad (5)$$

where $f(r)$ once more is a positively defined damping function.

The Hilbert transform $\hat{x}(t)$ of the process $x(t)$ may be approximately written as

$$\hat{x}(t)\approx-\frac{\dot{x}(t)}{\omega_0} \quad (6)$$

and its envelope in turn as

$$r(t)=\sqrt{x^2(t)+\hat{x}^2(t)}\approx\sqrt{x^2(t)+\frac{\dot{x}^2(t)}{\omega_0^2}}. \quad (7)$$

From mutual consideration of (6) and (7) we may come to the following system of two first order differential equations

$$\dot{x}\approx-\omega_0\sqrt{r^2-x^2}; \quad (8)$$

$$\dot{r}=\frac{\dot{x}}{r}\left(x+\frac{\ddot{x}}{\omega_0^2}\right). \quad (9)$$

With the help of (5) the brackets in (9) may be written as

$$x+\frac{\ddot{x}}{\omega_0^2}=\frac{\sqrt{K}\xi-f(r)\dot{x}}{\omega_0^2} \quad (10)$$

so, for the envelope $r(t)$ of the process $x(t)$ generated by SDE (5) we obtain the following differential equation

$$\dot{r}=-\frac{\sqrt{r^2-x^2}}{r\omega_0}\left[\sqrt{K}\xi+\omega_0\sqrt{r^2-x^2}f(r)\right]. \quad (11)$$

The system of differential equations (8) and (9) generates a vector Markovian process (a, r) defined by its partial drift $a_x(x, r)$, $a_r(x, r)$ and diffusion $b_x(x, r)$, $b_r(x, r)$ functions [8]

$$a_x(x, r)=-\omega_0\sqrt{r^2-x^2}; \quad (12)$$

$$a_r(x, r)=-\frac{r^2-x^2}{r}f(r)+\frac{N_0}{4\omega_0^2}\frac{x^2}{r^3}; \quad (13)$$

$$b_x(x, r)=0; \quad (14)$$

$$b_r(x, r)=\frac{N_0}{2\omega_0^2}\frac{r^2-x^2}{r^2}. \quad (15)$$

The Fokker — Planck equation (FPE) for the transition probabilities $P_{x,r}(x, r, t)$ is written as

$$\frac{\partial}{\partial t}P_{x,r}(x, r, t)=-\frac{\partial}{\partial x}\left[a_x(x, r)P_{x,r}(x, r, t)\right]-\\-\frac{\partial}{\partial r}\left[a_r(x, r)P_{x,r}(x, r, t)\right]+\\+\frac{1}{2}\frac{\partial^2}{\partial r^2}\left[b_r(x, r)P_{x,r}(x, r, t)\right]. \quad (16)$$

While the operator of the generating SDE (5) does not depend on time, the stationary PDFs $P_{x,r}(x, r)=\lim_{t\to\infty}P_{x,r}(x, r, t)$ exists and is written as [1]

$$P_{x,r}(x,\ r)=\frac{P_r(r)}{\pi\sqrt{r^2-x^2}}, \qquad (17)$$

where $P_r(r)$ is the stationary PDF of the r-component of the vector process (x, r). Substituting expressions (12)–(15) in (16) we derive the following stationary FPE:

$$\frac{\partial}{\partial r}\left[\frac{\sqrt{r^2-x^2}}{r^2}P_r(r)\right]=$$

$$=-\frac{4\omega_0^2}{N_0}\frac{\sqrt{r^2-x^2}}{r}f_1(r)P_r(r)+\frac{P_r(r)}{\omega_0^2 r^3\sqrt{r^2-x^2}}. \qquad (18)$$

Integrating (18) respect to $x \in (-r,\ r)$ we obtain the expression for $f(r)$ as a function of the generating SDE parameters (N_0, ω_0) and the required envelope PDF $P_r(r)$ is

$$f(r)=\frac{N_0}{4\omega_0^2 r}\left[\frac{1}{r}-\frac{\mathrm{d}}{\mathrm{d}r}\ln P_r(r)\right]. \qquad (19)$$

For the Nakagami PDF of the envelope this function is written as

$$f(r)=\frac{N_0}{2\omega_0^2}\left(\frac{m}{\sigma^2}+\frac{1-m}{r^2}\right) \qquad (20)$$

and the SDE (5) itself, which generates the corresponding stationary pass band process takes the form

$$\ddot{x}+\frac{N_0}{2\omega_0^2}\left(\frac{m}{\sigma^2 r^{2(1-m)}}+\frac{1-m}{r^2}\right)\dot{x}+\omega_0^2 x=\sqrt{\frac{N_0}{2}}\xi. \qquad (21)$$

It is clear now that a process with a sub-Rayleigh fading ($0.5 \le m < 1$) is represented as a stationary reaction to WGN excitation of the passive oscillator with energy dependent damping. In the Rayleigh case the oscillator is linear. Evidently, the following equivalent equation, is more convenient for simulation application then (20) and may be written as

$$r^2\ddot{x}+\frac{N_0}{2\omega_0^2}\left(\frac{m}{\sigma^2}r^{2m}+1-m\right)\dot{x}+\omega_0^2 r^2 x=\sqrt{\frac{N_0}{2}}r^2\xi. \qquad (22)$$

In the case of Weibull PDF of the envelope, the nonlinear function in (4) is written as

$$f(r)=\frac{N_0}{4\omega_0^2}\left(\frac{2-\alpha}{r^2}+\alpha\beta r^{\alpha-2}\right) \qquad (23)$$

and the generating SDE is

$$r^2\ddot{x}+\frac{N_0}{4\omega_0^2}\left(2-\alpha+\alpha\beta r^{\alpha}\right)\dot{x}+\omega_0^2 r^2 x=\sqrt{\frac{N_0}{2}}r^2\xi. \qquad (24)$$

In the case $1 \le \alpha < 2$ (a sub-Rayleigh case) the oscillator corresponding to (24) is, once more, a passive one with the energy dependent damping.

Correlation Function of the Envelope of Sub-Rayleigh Fading Carrier

Let us return to (5) and consider instead of the vector process (x, r) another one (r, φ), where r is defined by (7) and

$$\varphi(t)=-\omega_0 t-\operatorname{arctg}\left(\frac{\dot{x}}{\omega_0 x}\right). \qquad (25)$$

Differentiation of (25) with respect to time gives

$$\dot{\varphi}=-\frac{x\left(\ddot{x}+\omega_0^2 x\right)}{\omega_0^2 r^2}. \qquad (26)$$

Substituting in the system of equations (7) and (26), defining together the process (r, φ), expression for $\ddot{x}$ from (5) and

$$\dot{x}=-\omega_0 r\sin\left(\omega_0 t+\varphi\right) \qquad (27)$$

we easily obtain equations generating $r(t)$ and $\varphi(t)$. Their simplification is based on the assumption of the effective filtering of high harmonics in the generating SDE solution. The corresponding procedure for the envelope $r(t)$ which interests us leads to the first-order SDE

$$\dot{r}=-\frac{C}{2}r+\frac{\omega_0^2 N_0}{8r}+\varsigma(t). \qquad (28)$$

In (28) $\zeta(t)$ is a WGN with unit spectral density. From (28) it is clear that the envelope $r(t)$ of the Gaussian narrow band process is Rayleigh distributed, i. e.

$$P_r(r)=\frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right), \qquad (29)$$

where

$$\sigma^2=\frac{\omega_0^2 N_0}{4C}. \qquad (30)$$

Since the results of eliminating the vibrations are not related to the specific form of $f(r)$ in (5), the envelope of the corresponding non-Gaussian process $x(t)$, just as in the linear case may be considered as a one-dimensional Markovian process with the diffusion function $b_r(r)$ independent of the PDF.

As the stationary PDF of the Markovian one-dimensional process with the diffusion function constant and equal to unity is defined as [8]

$$P_r(r) = \text{const}\exp\left[-\int a_r(r)\mathrm{d}r\right], \quad (31)$$

its drift function is

$$a_r(r) = \frac{\mathrm{d}}{\mathrm{d}r}\ln P_r(r), \quad (32)$$

and so in the case of Nakagami $P_r(r)$ the corresponding SDE may be written as

$$\dot{r} = -\frac{2m-1}{r} + \frac{8Cmr}{\omega_0^2 N_0} + \varsigma_1(t). \quad (33)$$

The SDE (33) generates one of the few Markovian processes for which the exact expression for the correlation function can be obtained. It is written as [8]

$$B_r(\tau) = \frac{\omega_0^2 N_0 \Gamma^2(m+0.5)}{16C\pi m\Gamma(m)} \times$$
$$\times \sum_{i=1}^{\infty} \frac{\Gamma^2(i-0.5)}{\Gamma(i+m)i!}\exp\left(-i\frac{8Cm}{\omega_0^2 N_0}|\tau|\right). \quad (34)$$

For the sub-Rayleigh envelope PDF ($m < 1$) this expression is well approximated by the first member of the expansion:

$$B_r(\tau) \approx \frac{\omega_0^2 N_0 \Gamma^2(m+0.5)}{16Cm^2\Gamma^2(m)}\exp\left(-\frac{8Cm}{\omega_0^2 N_0}|\tau|\right), \quad (35)$$

which with the help of (30) may be rewritten as

$$B_r(\tau) \approx \frac{\sigma^2\Gamma^2(m+0.5)}{4m^2\Gamma^2(m)}\exp\left(-\frac{2m}{\sigma^2}|\tau|\right). \quad (36)$$

Thus the correlation function of the fading carrier envelope is rather close to the exponent (a widely used model of the fading correlation). Formally, from (36) it appears as if the correlation interval of a sub-Rayleigh fading exceeds the same for the Rayleigh one, i. e. sub-Rayleigh fading is slower than the latter. This interpretation, nevertheless, ignores the fact that an increase of the channel fading depth practically always leads to a decrease of the envelope mean value

$$\bar{r} = \frac{\sigma\Gamma(m+0.5)}{\sqrt{m}\Gamma(m)}, \quad (37)$$

which depends slightly on m if $0.5 < m \le 1$. So the required decrease of the envelope PDF parameter σ may be obtained by the decline of N_0 in (30) and (5) which, in turn, leads to decreasing of the sub-Rayleigh envelope correlation interval, and as a result to its approximate independence on the parameter m value.

Unfortunately, in the case of the Weibull envelope PDF the corresponding Fokker — Planck equation cannot be solved analytically and due this it is impossible to obtain closed expression for the Weibull fading correlation function.

Conclusions

This paper has addressed the problem of modeling fading in situations where the Gaussian assumption no longer applies. The received fading signal is represented by the corresponding nonlinear oscillator (generating system) excited by WGN. The approach is quite different from the one, which generates the fading process as a sum of sinusoids [9] as well as from its representation as a product of a certain baseband and complex Gaussian processes [3]. Its main advantage is that the marginal PDF of the signal envelope and its correlation function are easy controlled. In particular, the former can be accommodated by a suitable choice of the spectral density of the WGN and of the energy dependent oscillator damping. With regard to simulation of non-Rayleigh fading the procedure of the corresponding oscillator synthesis is presented. In the sub-Rayleigh case (Nakagami-$m < 1$, and Weibull-$\alpha < 2$) the oscillator appears to be a passive one. The correlation function of the presented model envelope is analyzed. It is rather close to an exponent, but depends on the fading depth.

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Модель глубокого фединга

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Введение: качество приема радиосигнала в существенной степени зависит от диапазона случайных изменений его уровня, то есть от глубины фединга. Иногда распределение огибающей принимаемого сигнала оказывается сдвинутым влево относительно закона Рэлея. Радиоканал с таким распределением фединга может рассматриваться как «критический». **Цель:** синтез модели узкополосного случайного процесса с распределением огибающей, сдвинутого влево относительно закона Рэлея. **Результаты:** процедура синтеза модели основана на представлении процесса как реакции устойчивой динамической системы, возбуждаемой белым гауссовым шумом. Получены нелинейные устойчивые стохастические дифференциальные уравнения второго порядка для моделирования замирающей несущей радиосигнала с огибающей, распределенной по законам Накагами или Вейбулла. Огибающая рассматривается как марковский непрерывный процесс. Получено аналитическое выражение для корреляционной функции. Показано, что по крайней мере в случае замираний по Накагами интервал корреляции огибающей почти не зависит от глубины замираний. **Практическая значимость:** в различных аппликациях, включая радиосвязь в закрытых помещениях, уровень замирающего сигнала в течение значительных временных интервалов критически низок. Предложенная модель, используемая в качестве основного блока имитатора такого радиоканала, позволяет осуществить оценку качества системы связи на уровне ее проектирования.

Ключевые слова — рэлеевские замирания, марковские диффузионные процессы, стохастические дифференциальные уравнения, распределение Накагами огибающей, распределение Вэйбулла огибающей, модель распространения радиосигнала.

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