UDC 517.977
doi:10.31799/1684-8853-2020-2-71-77

Articles

Utility function in the foraging problem with imperfect information

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Introduction: *One of the foraging theory problems is choosing the most suitable patch as a source of energy (food) resources for the population. A promising approach to study this problem is based on the Boltzmann distribution. In statistical physics, the Boltzmann distribution describes the probability of a system falling into a particular energy state.* ***Purpose:*** *The development of this approach in order to solve the patch selection problem. The solution based on the utility functions should be used to construct the probability distribution.* ***Methods:*** *Construction and analysis of the patch utility function which takes into account the time and population movement. Based on utility functions, domains are built which characterize the probability of choosing a patch. Boltzmann distribution is used to specify the patch selection probabilities.* ***Results:*** *A utility function depending on time is proposed and analyzed. A measure of the population's awareness of the patch suitability is proposed, which depends on the distance to the patch at a given time. The utility function properties have been investigated. The influence of its information component on the patch selection process is analyzed. The patches are classified as "bad" or "good" according to the amount of food resources they contain. The study showed that a population may choose a bad patch on a certain time interval. Preferential utility domains are constructed and their kinematics is analyzed.* ***Particular relevance:*** *The results obtained allow you to forecast the behavior of a population choosing a suitable patch.*

Keywords *— utility function, preferential utility domain, measure of awareness.*

For citation: Kirillov A. N., Danilova I. V. Utility function in the foraging problem with imperfect information. *Informatsionno-upravliaiushchie sistemy* [Information and Control Systems], 2020, no. 2, pp. 71–77. doi:10.31799/1684-8853-2020-2-71-77

Introduction

The problem of selection of the patch by the population, that is most suitable for consumption of the resources contained in it, is studied in the theory of optimal foraging. It is assumed that the population, in search of a suitable patch, acts in such a way as to maximize the amount of consumed energy [1–5].

V. Krivan developed the concept of ideal free distribution [6, 7] to solve the problem of optimal selection of suitable patch. According to the ideal free distribution the population has perfect information about the quality of patches and it is distributed between patches so as to maximize an energy consumption rate. He showed that the ideal free distribution in the problem of selection of the patch by the population is evolutionarily stable [6]. Empirical observations show that the ideal free distribution model is not adequate to the real processes for selection of the patch, because poor patches also attract individuals of the population. In addition, the population does not have perfect information on the quality of patches. Influence of migration on the stability behavior of a single-species population is studied in [8].

A game-theoretic approach to the solution of the problem of optimal selection of a suitable patch by the population was proposed in [9]. The Nash equilibrium is used as a criterion of optimality.

The concept of information, applicable to the theory of optimal foraging, has been widely used in neurobiology. The decision-making by the population on the selection of a strategy during a search for food with taking into account the accumulation and loss of data over time was studied in [10–13]. In [5, 14, 15] it was shown that food search strategies and the transition between them is explained by an increase in awareness. In [16], an elementary heuristic, which underlies the behavior of a population during a search, is considered; it is shown on the basis of experimental data that a search must be considered as an unsteady process that transfers an organism from one information state to another. A model of data accumulation for a population, that makes a decision to leave the patch, is analyzed in [17, 18]. This analysis is based on the marginal value theorem [19]. In [20, 21], the issue of selection of the patch is considered with taking into account the density of the population that selected this patch. In [22], a negative and positive relationship between the energy capacity of the patch and the frequency of visits by population is analyzed.

U. Dieckmann [23] proposed the following approach to solve the problem of selection of the patch. This approach is based on the utility function that takes into account the cost of moving to patch and the measure of awareness of its quality. In addition, the population is situated in one of the patches and evaluates the usefulness of other patches.

On the basis of [23], a utility function which takes into account population movements between patches, i. e., a change in its position depending on time, was proposed in [24]. The notion of preferential utility domain (PUD) of a patch was proposed in [24]. If the population locates in some PUD, it selects the patch, that also locates in this domain, with more probability than any patch located in other domain.

This work develops investigations presented in [24]. A measure of the population's awareness of the suitability of patch is proposed. It depends on the varying distance to patch and on the current time as well. The properties of the utility function are investigated and the PUDs are constructed. That takes into account the current time and average awareness of the quality of patches. The PUD partition of the environment, in which the population moves, depends on time, that makes it possible for the worst patch to be included in the PUD of the best patch at some time, evaluated in this paper.

The utility function, that is proposed in this paper, is used to determine the probability of selection of the patch by a population at any time. The Boltzmann distribution [23] is used as the probability of selection of patch. In statistical physics, the Boltzmann distribution usually describes of the probabilities of system energy states.

The utility function and its properties

The following problem is considered in [23]. A population is located in the patch i, containing some energy resource, and can move to patch $j \neq i$. The probability of moving from i to j is determined by the Boltzmann distribution: $P_{ij} = \frac{e^{qU_{ij}}}{\sum_{j=1}^{m} e^{qU_{ij}}}$, where m is the number of patches; U_{ij} is the utility of patch j for a population, located in patch i:

$$U_{ij} = V_j I_{ij} + (1 - I_{ij})\overline{V} - T_{ij},\ i,\ j = 1,\ ..,\ m, \qquad (1)$$

where I_{ij} is a measure of the awareness of a population, locating in the patch i, about the patch j. Let $I_{ij} \in [0,\ 1]$, where $I_{ii} = 1$, i. e. the population has complete information about the patch in which it is located; V_j is the amount of food resources in the patch j; $\overline{V} = \gamma_1 V_1 + \gamma_2 V_2 + \ldots + \gamma_m V_m$ is the average utility of the patches for a population located in patch i, $\gamma_1 + \gamma_2 + ... + \gamma_m = 1$, $\gamma_i \geq 0$; T_{ij} is a function of the cost of moving from i to j.

Unlike [23], where the population is static, in the present paper, the population moves in the space between patches. In addition, the movement is not determined, because according to ecological investigations, it is almost impossible to propose a motion model even for a certain species. We introduce the utility function $U_i(d_i,\ t)$ of the patch i at time t for population, located at a distance d_i from the patch i:

$$U_i(d_i,\ t) = V_i I_i(d_i,\ t) + (1 - I_i(d_i,\ t))\overline{V} - T_i(d_i),$$
$$i = 1,\ ..,\ m, \qquad (2)$$

where $I_i(d_i,\ t)$ is a population measure of the awareness of the patch i, $I_i(d_i,\ t) \in [0,\ 1]$, $\overline{V} = \gamma_1 V_1 + \gamma_2 V_2 + \ldots + \gamma_m V_m$ is the average utility of the patches for a population, $\gamma_1 + \gamma_2 + ... + \gamma_m = 1$, where $\gamma_i = \text{const}$, $\gamma_i \geq 0$; $T_i(d_i)$ is a function of the cost of moving to i, we will assume that: $\frac{\partial T_i}{\partial d_i} \geq 0.$ From (2), it follows that the structure of the utility function U_i substantially depends on the information component that is represented by the first two terms. We assume that the measure of awareness $I_i(d_i,\ t)$ has the following properties. The population has complete information on the patch in which it is located: $I_i(0,\ t) = 1$, for any $t \in \mathbb{R}$.

The farther the population is located from a patch i, the less information on the value V_i it has, i. e. $\frac{\partial I_i}{\partial d_i} < 0,\ \lim_{d_i \to \infty} I_i(d_i,\ t) = 0.$ If $d_i \neq 0$, then as time t increases, the population's awareness of the patch i increases: $\frac{\partial I_i}{\partial t} > 0,\ \lim_{t \to \infty} I_i(d_i,\ t) = 1.$

Basing on the foregoing, we formulate the properties of the utility function U_i. We assume that the patch i is "good" if $V_i > \overline{V}$, and it is "bad" if $V_i < \overline{V}$.

1. *With the time increase, the usefulness of the "bad" patch decreases, and the usefulness of the "good" patch increases.* Indeed, taking into account (2), we have: $\frac{\partial U_i}{\partial t} = \frac{\partial I_i}{\partial t}(V_i - \overline{V})$. Since $\frac{\partial I_i}{\partial t} > 0$, then $\frac{\partial U_i}{\partial t} > 0$ for $V_i > \overline{V}$, and $\frac{\partial U_i}{\partial t} < 0$ for $V_i < \overline{V}$. Thus, the longer the time of study of the "bad" patch by population, the less the attractivity of it for population, and vice versa.

2. *If $V_i > \overline{V}$, then its utility of the i-th patch decreases with the increasing of d_i.*

Then: $\frac{\partial U_i}{\partial d_i} = \frac{\partial I_i}{\partial d_i}(V - \overline{V}) - \frac{\partial T_i}{\partial d_i} < 0$, that follows from the properties I_i and T_i.

Let us consider in more detail the structure of the utility function U_i. Value $W_i = V_i I_i + (1 - I_i)\overline{V}$ is the information component of the utility function U_i. Thus, the measure of awareness I_i, W_i do not influence the property 2, since $\frac{\partial W_i}{\partial d_i} < 0$ for any value I_i.

3. *If $V_i < \overline{V}$, then its utility of the i-th patch may decrease or increase while d_i increases.* Since, the sign of the derivative $\frac{\partial U_i}{\partial d_i}$ coincides with the sign

of the difference $\frac{\partial I_i}{\partial d_i}(V-\overline{V})-\frac{\partial T_i}{\partial d_i}$ which depends on t, then the information component W_i considerably influences the sign of the derivative $\frac{\partial U_i}{\partial d_i}$.

Thus, in the case of a "bad" patch, i. e. for a small value V_i, $V_i<\overline{V}$, information component W_i plays a more significant role in the process of the patch selection, basing of the utility function $U_i(d_i, t)$, than in the case of a "good" patch V_i. In other words, a "good" patch is more noticeable than a "bad" at long distances.

Example. Assume $I_i=e^{-\frac{\beta d_i^2}{t+C}}$, $T_i(d_i)=\alpha d_i^2$, where α, β, C are the positive constants. Here β is the coefficient of forgetting. It was shown in [24] that with increasing of β the population measure of awareness, I_i, decreases. Thus, the larger the value of β, the faster the population forgets information on the patch; C is the constant that determines the measure of awareness of the patch the initial time instant $t=0$; α is the coefficient of cost, $\overline{V}=\gamma_1 V_1+\gamma_2 V_2+\ldots+\gamma_m V_m$, $\gamma_i \geq 0$, $\gamma_1+\gamma_2+\ldots+\gamma_m=1$. Then, the utility function has the following form:

$$U_i(d_i, t)=V_i e^{\frac{-\beta d_i^2}{t+C}}+\left(1-e^{\frac{-\beta d_i^2}{t+C}}\right)\overline{V}-\alpha d_i^2. \quad (3)$$

Since $\frac{\partial U_i}{\partial t}=\frac{\beta d_i^2}{(t+C)^2}e^{\frac{-\beta d_i^2}{t+C}}(V_i-\overline{V})$, then, obviously, the property 1 is satisfied. It is easy to show that $\frac{\partial U_i}{\partial d_i}=2d_i(\overline{V}-V_i)\frac{\beta}{t+C}e^{\frac{-\beta d_i^2}{t+C}}-2d_i\alpha$, whence it follows that the property 2 is also satisfied. In the case of a "bad" patch, $V_i<\overline{V}$, it follows from the form of derivative $\frac{\partial U_i}{\partial d_i}$, that its sign substantially depends on the measure of awareness $I_i=e^{-\frac{\beta d_i^2}{t+C}}$ and its derivative $\frac{\partial I_i}{\partial d_i}$.

Two-dimensional case

Consider the case $m=2$. For convenience, denote the first and second patches by A_1 and A_2, respectively. Utility functions for two patches are: $U_i(d_i, t)=V_i e^{\frac{-\beta d_i^2}{t+C}}+(1-e^{\frac{-\beta d_i^2}{t+C}})\overline{V}-\alpha d_i^2$, $i=1, 2$, where $\overline{V}=\gamma_1 V_1+\gamma_2 V_2$. Let $V_1<V_2$, i. e. the second patch is more attractive than the first.

Consider, in more detail, the case of a "bad" patch, for $m=2$. Let the patch $i=1$ be "bad", i. e. $V_1<V_2$, $V_1<\overline{V}=\gamma_1 V_1+\gamma_2 V_2$. Taking into account the form of the function $U_i(d_i, t)$ (3), it is easy to show that the following statement is valid.

Proposition 1. Assume $d_1^2>\gamma_2(V_2-V_1)/ae$. Then $\frac{\partial U_1}{\partial d_1}<0$ for any $t\geq 0$.

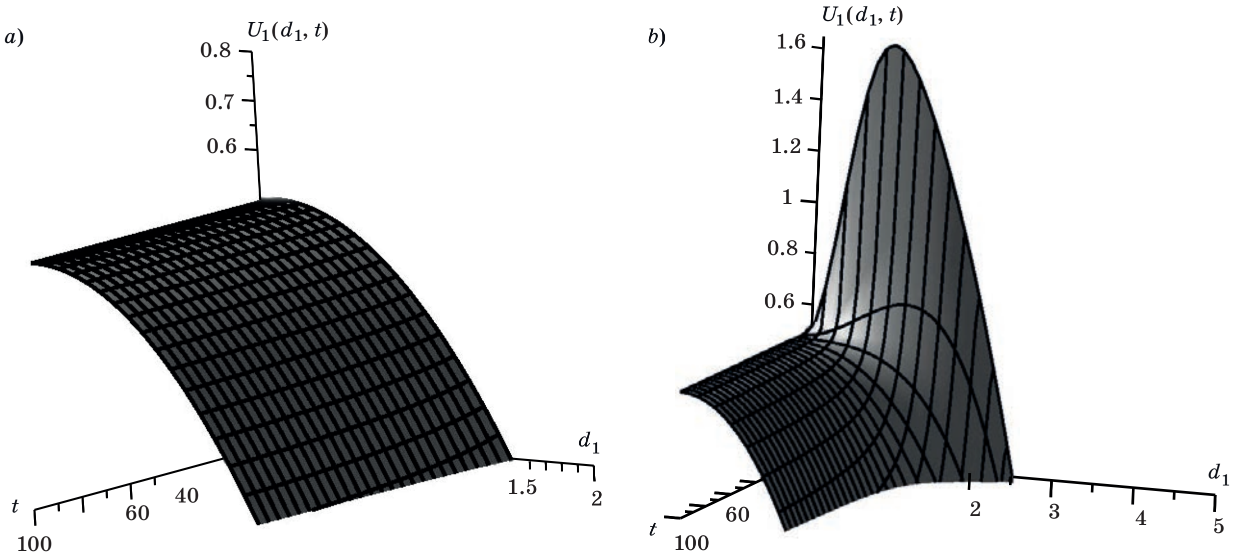
Proposition 2. Assume $d_1^2<\gamma_2(V_2-V_1)/ae$. Then there exist $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{t}_1<\tilde{t}_2$, such that $\frac{\partial U_1(d_1, t)}{\partial d_1}<0$ for $t\in[0, \tilde{t}_1)\cup(\tilde{t}_2, +\infty)$, $\frac{\partial U_1(d_1, t)}{\partial d_1}>0$ for $t\in[\tilde{t}_1, \tilde{t}_2]$.

Proof. Two cases are possible: $e^{\frac{-\beta d_1^2}{(t+C)}}<\frac{\alpha(t+C)}{\beta\gamma_2(V_2-V_1)}$ or $e^{\frac{-\beta d_1^2}{(t+C)}}>\frac{\alpha(t+C)}{\beta\gamma_2(V_2-V_1)}$, or, equivalently, $\frac{\beta}{t+C}e^{\frac{-\beta d_1^2}{t+C}}<\frac{\alpha}{\gamma_2(V_2-V_1)}$ or $\frac{\beta}{t+C}e^{\frac{-\beta d_1^2}{t+C}}\geq\frac{\alpha}{\gamma_2(V_2-V_1)}$.

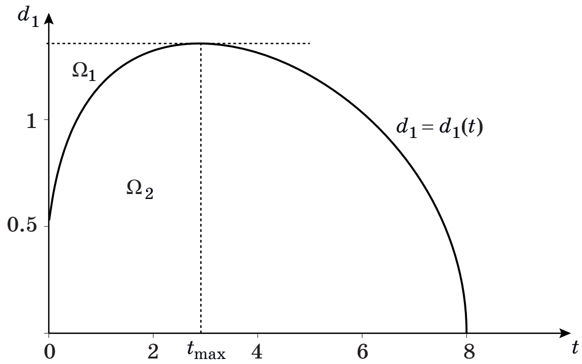
Denote $z=\frac{\beta}{t+C}$, $A=\frac{\gamma_2(V_2-V_1)}{\alpha}$, $f(z)=ze^{-zd_1^2}$. It's obvious that $z^*=\frac{1}{d_1^2}$ is a point of maximum of $f(z)$ and $f(z^*)=\frac{1}{ed_1^2}$ is the maximum value of $f(z)$. It follows from the assumption of the Proposition 2 that $\frac{1}{ed_1^2}>\frac{1}{A}$. Then $e^{-zd_1^2}<\frac{1}{Az}$ for $z<\tilde{z}_1$ or $z>\tilde{z}_2$, and $e^{-zd_1^2}>\frac{1}{Az}$ for $z\in[\tilde{z}_1, \tilde{z}_2]$, where $\tilde{z}_1=\frac{\beta}{\tilde{t}_2+C}$, $\tilde{z}_2=\frac{\beta}{\tilde{t}_1+C}$ for some values $t=\tilde{t}_1$ and $t=\tilde{t}_2$. Thus, $\frac{\partial U_1}{\partial d_1}<0$ for $t<\tilde{t}_1$, or $t>\tilde{t}_2$, $\frac{\partial U_1}{\partial d_1}>0$ for $t\in[\tilde{t}_1, \tilde{t}_2]$.

The proposition 2 is proved.

The form of the utility function $U_1=U_1(d_1, t)$, corresponding to the Propositions 1 and 2, is shown in Fig. 1, *a* and *b*, respectively. It should be noted that the case of a small distance of the population from the "poor" patch is considered in the Proposition 2. Unlike a "good" patch 2, a "bad" patch 1 is a weak source of information. Thus, the process of estimation of the utility of a "bad" patch has some uncertainty for sufficiently small t. Consider the domain $\Omega=\Omega_1\cup\Omega_2$, where $\Omega_1=$



■ ***Fig. 1.*** Utility function $U_1(d_1, t)$ of patch A_1: $\gamma_1 = 0.4$, $\gamma_2 = 0.6$, $V_1 = 0.5$, $V_2 = 3$, $\alpha = 0.3$, $\beta = 1.6$; *a* — $C = 50$; *b* — $C = 0.5$



■ ***Fig. 2.*** Domains Ω_1, Ω_2: $\gamma_1 = 0.4$, $\gamma_2 = 0.6$, $V_1 = 0.5$, $V_2 = 3$, $\alpha = 0.3$, $\beta = 1.6$, $C = 0.5$

$= \left\{(d_1, t): \frac{\partial U_1}{\partial d_1} > 0, d_1 \ge 0, t \ge 0\right\}$, $\Omega_2 = \left\{(d_1, t): \frac{\partial U_1}{\partial d_1} < 0, (d_1, t) \le d_1(t_{\max}), t \in [0, t_{\max}]\right\}$. For sufficiently small t and d_1, i. e. when points $(t, d_1) \subset \Omega$, derivative $\frac{\partial U_1}{\partial d_1}$ can change its sign depending on t, d_1 and the relations between the parameters α, β, V_1, $\overline{V}$, C. Consider the equality $\frac{\partial U_1}{\partial d_1} = 0$. After simple transformations, it implies that $d_1(t) = \sqrt{\frac{t + C}{\beta} \ln \frac{\gamma_2 \beta (V_2 - V_1)}{\alpha (t + C)}}$. The function $d_1(t)$ is continuous and has a unique maximum at $t = t_{\max}$, where $t_{\max} \in (0, \tilde{t})$, $\tilde{t} = \frac{\gamma_2 (V_2 - V_1) \beta}{\alpha e} - C$ (Fig. 2).

Domains of preferred utility

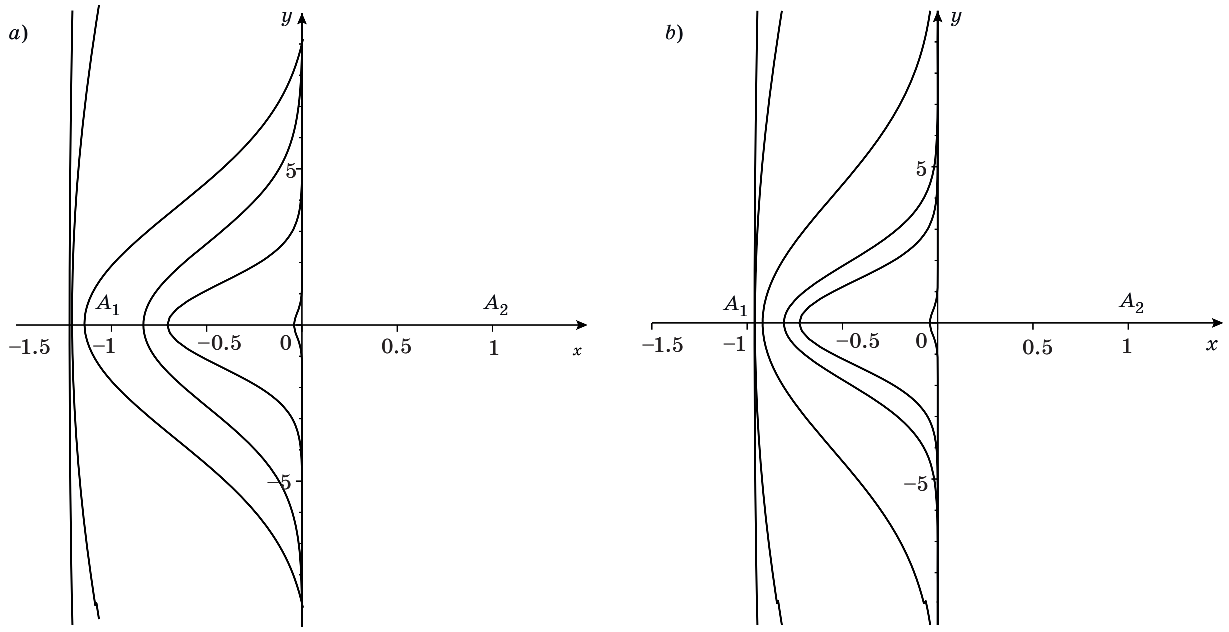
In [24], the definition of PUD was introduced and their common boundary was found. Taking into account that in the present paper, unlike [24], utility also depends on t, we propose the following definition of the PUD.

Consider the patch i as a point $A_i \in \mathbb{R}^n$. Assume that a point $M(t) = M \in \mathbb{R}^n$ corresponds to a population position at a time t, $d_i(t) = \rho(M(t), A_i)$ — the distance of $M(t)$ from A_i.

Definition. The preferential utility domain $D_i(t)$ of the patch A_i is a set $D_i(t) = \{M(t) \in \mathbb{R}^n: \rho(M(t), A_i) = d_i(t), U_i(d_i(t), t) > U_j(d_j(t), t), i \ne j, j = 1, .., m\}$. Here $U_i(d_i(t), t)$ is the utility function (3).

It follows from the definition that the boundaries of the domains $D_i(t)$ change in time. Let us consider the asymptotic behavior of the partition of the space $\mathbb{R}^2 = \{(x, y): x, y \in \mathbb{R}\}$ into domains $D_i(t)$ for the case $m = 2$. Without loss of generality, we assume that the patches have the following coordinates: $A_1(-1, 0)$, $A_2(1, 0)$, $D_1(t)$, $D_2(t)$ are PUDs of patches A_1 and A_2, respectively. Denote $g(t) = \{M(t) \in \mathbb{R}^2: \rho(M(t), A_i) = d_i(t), U_1(d_1(t), t) = U_2(d_2(t), t), i = 1, 2\}$ the common boundary of $D_1(t)$, $D_2(t)$. Here $\rho(M(t), A_i)$ is the distance between M and A_i. Consider question of "capture" by PUD_2 of A_1, where PUD_2 is the preferential utility domain of A_2. Denote $E = \ln \frac{\gamma_1 (V_2 - V_1)}{\gamma_2 (V_1 - V_2) + 4\alpha}$. It is not difficult to prove the following proposition.

Proposition 3. For existence of time $t = t^* \ge 0$, such that $A_1 \in D_2(t)$ for all $t \ge t^*$ it is necessary and sufficient that $(V_2 - V_1)/4 > \alpha$. Wherein: $t^* = 4\beta/E - C$, if $\alpha > \gamma_2 (V_2 - V_1)/4$ and $C \le 4\beta/E$,



■ ***Fig. 3.*** Case of "capture" (*a*) and of absence of "capture" (*b*) of patch A_1: $\gamma_1 = 0.4$, $\gamma_2 = 0.6$, $V_1 = 0.5$, $V_2 = 3$, $\beta = 1.6$, $C = 0.5$; a — $\alpha = 0.55$; b — $\alpha = 0.65$

or $t^* = 0$, if $\alpha > \gamma_2 (V_2 - V_1)/4$ and $C > 4\beta/E$ or if $\alpha \le \gamma_2 (V_2 - V_1)/4$.

Figures 3, *a* and *b* show cases of "capture" of patch A_1 and absence of "capture" of patch A_1, respectively.

The probability of patch selection

The utility function $U_i(d_i, t)$ of a patch i is used to find the probability $P_i(t)$ of selection of a suitable patch, by population, at time t. According [23], we use the Boltzmann distribution $P_i(t) = \frac{e^{qU_i(d_i,t)}}{\sum_{i=1}^{m} e^{qU_i(d_i,t)}}$ to obtain $P_i(t)$, where $q > 0$ is some constant. In [24] the system of m nonlinear, ordinary, non-autonomous differential equation was obtained:

$$\begin{cases} \dot{P}_1 = qP_1(P_2\varphi_{12} + \ldots P_m\varphi_{1m}) \\ \dot{P}_2 = qP_2(P_1\varphi_{21} + \ldots P_m\varphi_{2m}) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots , \\ \dot{P}_m = qP_m\left(P_1\varphi_{m1} + \ldots P_{m-1}\varphi_{m,(m-1)}\right) \end{cases}$$

where $\varphi_{ij} = \dot{U}_i - \dot{U}_j$.

In [11], for $m = 2$, the Lyapunov stability of the Boltzmann distribution which is a particular solution of the above system was proved. Since the form of the above system is the same as in [24], the stability result is also valid. It means that the use of the Boltzmann distribution in practical applications is reasonable.

Conclusion

A utility function, used to determine the probability of a patch selection by a population is proposed. Developing the approach presented in [23], where the utility function was introduced for static population, in this paper the proposition of population is varied and, therefore, the utility function depends on time. The patches are classified according to the amount of food resources. The properties of a utility function are studied. Herewith, the particular attention is focused on the influence of a utility function component. The Boltzmann distribution is used as the probability of patch selection. Preferential utility domains are constructed and their kinematics, as $t \to \infty$, is analyzed.

In this work, a mathematical model, that reflects the main qualitative laws characterizing the process of selection of a suitable patch by the population, is proposed. Based on the proposed approach, relying on experimental observations, it possible to construct models for specific populations.

Acknowledgements

This paper was supported by the RFBR (18-01-00249a).

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УДК 517.977
doi:10.31799/1684-8853-2020-2-71-77

Функция полезности в задаче фуражирования с неполной информацией

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Введение: одной из задач теории фуражирования является выбор популяцией наиболее пригодного ареала (участка) как источника энергетических ресурсов (ресурсов питания). Ранее был предложен подход для исследования этой задачи, основанный на идее распределения Больцмана. В статистической физике распределение Больцмана описывает вероятность попадания системы в то или иное энергетическое состояние. **Цель:** развитие данного подхода для решения задачи выбора популяцией наиболее пригодного ареала. При этом решение, в основе которого лежит функция полезности, используется для построения вероятностного распределения. **Методы:** построение и анализ функции полезности ареала, учитывающей время и перемещение популяции. Построение на основе функций полезностей областей, характеризующих вероятность выбора ареала. Для описания вероятностей выбора популяцией ареала используется распределение Больцмана. **Результаты:** предложена и проанализирована функция полезности, зависящая от времени. Предложена мера информированности, характеризующая знание популяции об ареале и зависящая от расстояния до ареала в данный момент времени. Исследованы свойства функции полезности. Проведен анализ влияния информационной составляющей на процесс выбора популяцией ареала. В зависимости от объема пищевых ресурсов, которые в них содержатся, ареалы делятся на «плохие» и «хорошие». В результате исследования выяснилось, что плохой ареал может быть выбран популяцией на некотором промежутке времени. Построены области предпочтительной полезности ареалов при изменении времени и исследована их кинематика. **Практическая значимость:** полученные результаты позволяют прогнозировать поведение популяции при выборе наиболее пригодного ареала.

Ключевые слова — функция полезности, область предпочтительной полезности, мера информированности.

Для цитирования: Kirillov A. N., Danilova I. V. Utility function in the foraging problem with imperfect information. *Информационно-управляющие системы*, 2020, № 2, с. 71–77. doi:10.31799/1684-8853-2020-2-71-77
For citation: Kirillov A. N., Danilova I. V. Utility function in the foraging problem with imperfect information. *Informatsionno-upravliaiushchie sistemy* [Information and Control Systems], 2020, no. 2, pp. 71–77. doi:10.31799/1684-8853-2020-2-71-77