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Articles

Research on the effectiveness of continuous-discrete cubature Kalman filter distributional-robust modifications

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Introduction: *Usually there are some outliers (abnormal measurements) in observed data, and they can significantly affect the quality of the data processing. Many dynamic processes are described with stochastic nonlinear equations. Modern nonlinear filters that include the cubature Kalman filter, which deserves a special attention, cannot effectively process data containing abnormal measurements. One of the possible solutions to this problem is to use so-called robust methods that have good performance when one has to analyze data containing outliers. The paper deals with the common situations, when the considered process is actually continuous, but the observed data is taken discretely.* ***Purpose:*** *Identifying the most effective advanced robust modifications of the continuous-discrete cubature Kalman filter and giving the appropriate recommendations for their appliance.* ***Results:*** *Four modifications of the continuous-discrete cubature Kalman filter have been proposed based on the variational Bayesian and correntropy robust approaches to parameter estimation for stochastic processes. All the modifications have parameters with optimal values depending on both the selected mathematical model and the considered set of observations composing the sample. These values are determined numerically by minimizing the accumulated root mean square error on some grid. The research on the effectiveness of the proposed robust modifications has been carried out for the problem of tracking a space vehicle during its reentry into the atmosphere. The stochastic and the grouped outliers have been considered. Two most effective filters that have approximately equal qualities of estimation have been derived. The correntropy filter that has one configurable parameter can be recommended for practical using.* ***Practical relevance:*** *The identified most effective robust filter can be used for solving various applied problems related to the identification of stochastic nonlinear continuous-discrete systems.*

Keywords *— nonlinear filtering, cubature Kalman filter, outliers, maximum correntropy criterion, variational Bayesian estimation, stochastic continuous-discrete system.*

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Introduction

In many fields of science and engineering, various applied problems related to statistical estimation of stochastic process parameters take place (see [1–3]). The optimal solution to such problems in the case of linear stochastic models can be obtained by applying the Kalman filter and its square-root modifications [4, 5]. Due to the fact that many real processes are described with nonlinear equations, nonlinear filtering methods are of particular importance.

Until the middle of the 1990s, the main method for solving nonlinear filtering problems was the extended Kalman filter [6, 7]. This filter is based on the linearization over the time domain of the state and measurement models conducted in the neighborhood of a specially defined nominal trajectory. In addition to the estimation accuracy loss, it has to be taken into account that even if there are no significant nonlinearities, the right-hand side of the state and the measurement equations may be determined with cumbersome analytical formulas. This leads to the problem of Jacobian matrices correct computing in the case of linearized models. Nowadays, the unscented (UKF) and the cubature Kalman filter (CKF) gain significant popularity as they don't have the mentioned drawbacks.

The UKF proposed in 1995 [8] and developed in [9–11] assumes using the optimal set of determined sigma points for approximating the first and the second moments of the system state vector. In each specific case, the quality of the results obtained with the UKF depends on the optimal choice of the filter parameter values.

The CKF used in this paper overcomes this drawback. For discrete systems, it was developed in 2009 [12], and after that it was modified for continuous-discrete systems in [13, 14]. To derive the CKF, the third-degree cubature rule was used for numerical approximation of a special type of multidimensional probability integrals [15, 16]. In practical terms, it is important that (unlike the UKF) the CKF has an algebraically equivalent square-root modification that provides computational robustness.

In practice, there are often the cases when observed data contain some abnormal measurements. This can be caused by failures occurring while gathering and transmitting the measurements. In mathematical terms, this can be interpreted as a significant deviation of the actual distribution of measurement noise from the postulated one for the points corresponding to outliers. To overcome these difficulties, various robust methods can be applied (see [17–20]) including the variational Bayesian [21–23] and the correntropy approaches [24–26] used in this paper. These methods are particularly worthy to be highlighted.

This paper provides a comparative analysis of some advanced modern robust modifications of the continuous-discrete cubature Kalman filter (CD-CKF). The used modifications have been chosen on the basis of the results of studies performed in [27, 28] for linear nonstationary systems. Also the materials of some relevant publications on nonlinear robust filtering [29–32] have been taken into account. It should be emphasized that all the considered filters have been obtained by applying the corresponding discrete expressions to the continuous-discrete case. In addition, some robust modifications of the CD-CKF have been purposely derived from already known UKF modifications.

This paper is the first in the author's series of papers devoted to the construction of resistant to abnormal measurements and machine rounding errors nonlinear continuous-discrete filter. Only the first part of the specified task is considered.

Structural-probabilistic description of the model

Following [33], consider the state space model of a controlled and observed stochastic nonlinear continuous-discrete system

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{f}\left[\mathbf{x}(t),\ \mathbf{u}(t),\ t\right] + \mathbf{\Gamma}(t)\mathbf{w}(t),\ t \in [t_0,\ t_N];$$

$$\mathbf{y}(t_{k+1}) = \mathbf{h}\left[\mathbf{x}(t_{k+1}),\ \mathbf{u}(t_{k+1}),\ t_{k+1}\right] + \mathbf{v}(t_{k+1}),$$

$$k = \overline{0,\ N-1},$$

where $\mathbf{x}(\cdot)$ is the n-dimensional state vector; $\mathbf{u}(\cdot)$ is the r-dimensional predefined control vector; $\mathbf{w}(\cdot)$ is the p-dimensional process noise vector; $\mathbf{y}(\cdot)$ is the m-dimensional measurement vector; $\mathbf{v}(\cdot)$ is the m-dimensional measurement noise vector.

Suppose that

— the stochastic processes $\{\mathbf{w}(t),\ t \in [t_0,\ t_N]\}$ and $\{\mathbf{v}(t_{k+1}),\ k = 0,\ 1,\ \ldots,\ N-1\}$ are the white Gaussian noises, and

$$\mathrm{E}\left[\mathbf{w}(t)\right] = \mathbf{0},\ \ \mathrm{E}\left[\mathbf{w}(t)\mathbf{w}^{\mathrm{T}}(\tau)\right] = \mathbf{Q}(t)\delta(t-\tau);$$

$$\mathrm{E}\left[\mathbf{v}(t_{k+1})\right] = \mathbf{0},\ \ \mathrm{E}\left[\mathbf{v}(t_{k+1})\mathbf{v}^{\mathrm{T}}(t_{i+1})\right] = \mathbf{R}(t_{k+1})\delta_{ki},$$

$$\mathrm{E}\left[\mathbf{v}(t_{k+1})\mathbf{w}^{\mathrm{T}}(\tau)\right] = \mathbf{0},\ k,\ i = \overline{0,\ N-1},\ \tau \in [t_0,\ t_N]$$

(here and further $\mathrm{E}[\cdot]$ is the mathematical mean value operator; $\delta(t-\tau)$ is the Dirac delta function; δ_{ki} is the Kronecker symbol);

— the initial state $\mathbf{x}(t_0)$ has the normal distribution with the parameters

$$\mathrm{E}\left[\mathbf{x}(t_0)\right] = \bar{\mathbf{x}}(t_0),$$

$$\mathrm{E}\left\{\left[\mathbf{x}(t_0) - \bar{\mathbf{x}}(t_0)\right]\left[\mathbf{x}(t_0) - \bar{\mathbf{x}}(t_0)\right]^{\mathrm{T}}\right\} = \mathbf{P}(t_0),$$

and it is uncorrelated with $\mathbf{w}(t)$ and $\mathbf{v}(t_{k+1})$;

— the covariance matrices of the process noise, the measurement noise and the initial state are known, $\mathbf{R}(t_{k+1})$ and $\mathbf{P}(t_0)$ are positive-definite matrixes;

— the observed data $\{\mathbf{y}(t_{k+1}),\ k = 0,\ 1,\ \ldots,\ N-1\}$ contain outliers.

CD-CKF

To construct the CKF, the third-degree Gaussian cubature rule [15, 16] is used to compute probability integrals of the following type

$$\mathrm{E}\left[\mathbf{g}(\mathbf{x})\right] = \int_{\mathrm{R}^n} \mathbf{g}(\mathbf{x}) N(\mathbf{x} \mid \boldsymbol{\mu},\ \boldsymbol{\Sigma})\mathrm{d}\mathbf{x} \approx$$

$$\approx \frac{1}{2n}\sum_{i=1}^{2n} \mathbf{g}(\boldsymbol{\mu} + \mathbf{L}_{\Sigma}\xi_i),$$

where $\mathbf{g}(\mathbf{x})$ is some vector function; $N(\mathbf{x} \mid \boldsymbol{\mu},\ \boldsymbol{\Sigma})$ is the probability density function of the n-dimensional normal distribution with the mean $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$; $\mathbf{L}_{\Sigma}$ is the lower triangular Cholesky factor $\left(\mathbf{L}_{\Sigma}\mathbf{L}_{\Sigma}^{\mathrm{T}} = \boldsymbol{\Sigma}\right)$, the nodes of a cubature formula ξ_i are defined with the expression

$$\xi_i = \begin{cases} \sqrt{n}\mathbf{e}_i,\ i = \overline{1,\ n}; \\ -\sqrt{n}\mathbf{e}_{i-n},\ i = \overline{n+1,\ 2n}, \end{cases}$$

where $\mathbf{e}_i = \Big(0,\ \ldots,\ \underset{i-1}{0},\ \underset{i}{1},\ \underset{i+1}{0},\ \ldots,\ 0\Big)^{\mathrm{T}}$.

Nowadays, two versions of the CD-CKF exist. The first one (see [13]) is based on the Itô-Taylor expansion of the 1.5 order used to discretize the corresponding stochastic differential equation and on applying the discrete formulas of the CKF from [12] to the obtained system. It should be noted, that preliminary discretization requires manual adjusting the optimal number of points of division for each specific problem. This leads to the lack of flexibility of the first CD-CKF version. In this work, the other version of the CD-CKF from [14] is used. This version is more accurate and actual in practical terms (see [34]), and it also doesn't have the mentioned drawback.

The CD-CKF algorithm

Step 1. Initialize the initial state and the covariance:

$$\hat{\mathbf{x}}(t_0 \mid t_0) = \overline{\mathbf{x}}(t_0),\ \mathbf{P}(t_0 \mid t_0) = \mathbf{P}(t_0).$$

Execute in a loop for $k = \overline{0,\ N-1}$:

Step 2. Obtain $\hat{\mathbf{x}}(t_{k+1} \mid t_k)$ and $\mathbf{P}(t_{k+1} \mid t_k)$ by solving the system of differential equations for $t \in [t_k, t_{k+1}]$:

$$\frac{d\hat{\mathbf{x}}(t \mid t_k)}{dt} = \frac{1}{2n}\sum_{i=1}^{2n} \mathbf{f}\left[\hat{\mathbf{x}}(t \mid t_k) + \mathbf{S}(t \mid t_k)\xi_i,\ \mathbf{u}(t),\ t\right];$$

$$\frac{d\mathbf{P}(t \mid t_k)}{dt} = \frac{1}{2n}\sum_{i=1}^{2n} \mathbf{f}\left[\hat{\mathbf{x}}(t \mid t_k) + \mathbf{S}(t \mid t_k)\xi_i,\ \mathbf{u}(t),\ t\right]\times$$
$$\times \xi_i^{\mathrm{T}}\mathbf{S}^{\mathrm{T}}(t \mid t_k) + \frac{1}{2n}\sum_{i=1}^{2n}\mathbf{S}(t \mid t_k)\xi_i \times$$
$$\times \mathbf{f}^{\mathrm{T}}\left[\hat{\mathbf{x}}(t \mid t_k) + \mathbf{S}(t \mid t_k)\xi_i,\ \mathbf{u}(t),\ t\right] +$$
$$+ \boldsymbol{\Gamma}(t)\mathbf{Q}(t)\boldsymbol{\Gamma}^{\mathrm{T}}(t),$$

where $\mathbf{S}(t \mid t_k) = \mathrm{Chol}[\mathbf{P}(t \mid t_k)]$ is the Cholesky factor for the matrix $\mathbf{P}(t \mid t_k)$.

Step 3. Calculate the cubature points and the propagated cubature points:

$$\mathbf{S}(t_{k+1} \mid t_k) = \mathrm{Chol}\left[\mathbf{P}(t_{k+1} \mid t_k)\right];$$

$$\chi_i(t_{k+1} \mid t_k) = \hat{\mathbf{x}}(t_{k+1} \mid t_k) + \mathbf{S}(t_{k+1} \mid t_k)\xi_i,\ i = \overline{1,\ 2n};$$

$$\gamma_i(t_{k+1} \mid t_k) = \mathbf{h}\left[\chi_i(t_{k+1} \mid t_k),\ \mathbf{u}(t_{k+1}),\ t_{k+1}\right],\ i = \overline{1,\ 2n}.$$

Step 4. Find the extrapolated measurement estimate and the update vector:

$$\hat{\mathbf{y}}(t_{k+1} \mid t_k) = \frac{1}{2n}\sum_{i=1}^{2n}\gamma_i(t_{k+1} \mid t_k);$$

$$\boldsymbol{\varepsilon}(t_{k+1}) = \mathbf{y}(t_{k+1}) - \hat{\mathbf{y}}(t_{k+1} \mid t_k).$$

Step 5. Obtain the cross-covariance matrix:

$$\mathbf{P}_{xy}(t_{k+1} \mid t_k) = \frac{1}{2n}\sum_{i=1}^{2n}\left[\chi_i(t_{k+1} \mid t_k) - \hat{\mathbf{x}}(t_{k+1} \mid t_k)\right]\times$$
$$\times\left[\gamma_i(t_{k+1} \mid t_k) - \hat{\mathbf{y}}(t_{k+1} \mid t_k)\right]^{\mathrm{T}}.$$

Step 6. Evaluate the covariance matrix of the prediction error:

$$\mathbf{P}_{yy}(t_{k+1} \mid t_k) = \frac{1}{2n}\sum_{i=1}^{2n}\left[\gamma_i(t_{k+1} \mid t_k) - \hat{\mathbf{y}}(t_{k+1} \mid t_k)\right]\times$$
$$\times\left[\gamma_i(t_{k+1} \mid t_k) - \hat{\mathbf{y}}(t_{k+1} \mid t_k)\right]^{\mathrm{T}} + \mathbf{R}(t_{k+1}). \quad (1)$$

Step 7. Calculate the Kalman gain factor:

$$\mathbf{K}(t_{k+1}) = \mathbf{P}_{xy}(t_{k+1} \mid t_k)\mathbf{P}_{yy}^{-1}(t_{k+1} \mid t_k). \quad (2)$$

Step 8. Obtain the filtered state estimate:

$$\hat{\mathbf{x}}(t_{k+1} \mid t_{k+1}) = \hat{\mathbf{x}}(t_{k+1} \mid t_k) + \mathbf{K}(t_{k+1})\boldsymbol{\varepsilon}(t_{k+1}). \quad (3)$$

Step 9. Find the corresponding error covariance matrix:

$$\mathbf{P}(t_{k+1} \mid t_{k+1}) = \mathbf{P}(t_{k+1} \mid t_k) - \mathbf{K}(t_{k+1})\times$$
$$\times\mathbf{P}_{yy}(t_{k+1} \mid t_k)\mathbf{K}^{\mathrm{T}}(t_{k+1}). \quad (4)$$

End of the loop for k.

In order to make the further discussion brief, when describing the robust modifications of the CD-CKF we will consider this algorithm to be a basis, but will add and edit certain steps.

Robust modifications of the CD-CKF

First of all, consider the continuous-discrete variational Bayesian-based cubature Kalman filter (CD-VBCKF) proposed in [29] and derived based on [22]. This modification suggests using the following parameters: ν_0 (scalar), $\mathbf{V}_0$ (m-dimensional square matrix), L (number of iterations for the filtration step), ρ (scaling factor selected on the interval (0, 1]). The optimal values of the parameters are determined by selection.

The CD-VBCKF algorithm

Step 1. At the step 1 of the CD-CKF algorithm, the initialization of the following parameters is included:

$$\upsilon(t_0 \mid t_0) = \upsilon_0,\ \mathbf{V}(t_0 \mid t_0) = \mathbf{V}_0.$$

Execute in a loop for $k = \overline{0,\ N-1}$:

Step 2. At the step 2 of the CD-CKF algorithm, the parameter calculating is included

$$\upsilon(t_{k+1} \mid t_k) = \rho\left(\upsilon(t_k \mid t_k) - n - 1\right) + n + 1;$$

$$\mathbf{V}(t_{k+1} \mid t_k) = \rho\mathbf{V}(t_k \mid t_k).$$

Steps 3–5 are equivalent to the steps 3–5 of the CD-CKF algorithm.

Step 6. Obtain

$$\mathbf{V}^0(t_{k+1} \mid t_{k+1}) = \mathbf{V}(t_{k+1} \mid t_k);$$

$$\upsilon(t_{k+1} \mid t_{k+1}) = 1 + \upsilon(t_{k+1} \mid t_k).$$

Execute in a loop for $j = \overline{1,\ L}$:

Step 7. Obtain

$$\mathbf{R}^j(t_{k+1}) = \left(v(t_{k+1} \mid t_{k+1}) - n - 1\right)^{-1} \mathbf{V}^{j-1}(t_{k+1} \mid t_{k+1}).$$

Step 8. Calculate $\mathbf{P}_{yy}^j(t_{k+1} \mid t_k)$ and $\mathbf{K}^j(t_{k+1})$ using the formulas (1), (2), where the matrices $\mathbf{R}(t_{k+1})$ and $\mathbf{P}_{yy}(t_{k+1} \mid t_k)$ are replaced with $\mathbf{R}^j(t_{k+1})$ and $\mathbf{P}_{yy}^j(t_{k+1} \mid t_k)$ respectively.

Step 9. Calculate $\hat{\mathbf{x}}^j(t_{k+1} \mid t_{k+1})$ and $\mathbf{P}^j(t_{k+1} \mid t_{k+1})$ by replacing $\mathbf{K}(t_{k+1})$ and $\mathbf{P}_{yy}(t_{k+1} \mid t_k)$ in the formulas (3) and (4) with the matrices $\mathbf{K}^j(t_{k+1})$ and $\mathbf{P}_{yy}^j(t_{k+1} \mid t_k)$ respectively.

Step 10. Calculate

$$\mathbf{S}^j(t_{k+1} \mid t_{k+1}) = \mathrm{Chol}\left[\mathbf{P}^j(t_{k+1} \mid t_{k+1})\right].$$

$$\chi_i^j(t_{k+1} \mid t_{k+1}) = \hat{\mathbf{x}}^j(t_{k+1} \mid t_{k+1}) + \mathbf{S}^j(t_{k+1} \mid t_{k+1})\xi_i;$$

$$\gamma_i^j(t_{k+1} \mid t_{k+1}) = \mathbf{h}\left[\chi_i^j(t_{k+1} \mid t_{k+1}),\ \mathbf{u}(t_{k+1}),\ t_{k+1}\right],$$

$$i = \overline{1,\ 2n}.$$

Step 11. Obtain the matrix

$$\mathbf{V}^j(t_{k+1} \mid t_{k+1}) = \frac{1}{2n}\sum_{i=1}^{2n}\left[\mathbf{y}(t_{k+1}) - \gamma_i^j(t_{k+1} \mid t_{k+1})\right] \times$$

$$\times \left[\mathbf{y}(t_{k+1}) - \gamma_i^j(t_{k+1} \mid t_{k+1})\right]^{\mathrm{T}} + \mathbf{V}(t_{k+1} \mid t_k).$$

End of the loop for j.

Step 12. Obtain

$$\mathbf{V}(t_{k+1} \mid t_{k+1}) = \mathbf{V}^L(t_{k+1} \mid t_{k+1}).$$

Step 13. Obtain the filtering estimate and the corresponding covariance matrix:

$$\hat{\mathbf{x}}(t_{k+1} \mid t_{k+1}) = \hat{\mathbf{x}}^L(t_{k+1} \mid t_{k+1});$$

$$\mathbf{P}(t_{k+1} \mid t_{k+1}) = \mathbf{P}^L(t_{k+1} \mid t_{k+1}).$$

End of the loop for k.

Now consider the correntropy modifications of the CD-CKF, which have been intensively developed in the recent years. Initially, the correntropy filters were obtained for linear dynamic models (see [35–37]). Later, they were successfully adapted for solving nonlinear problems [30–32, 38–40].

The correntropy filters are constructed on the basis of the maximum correntropy criterion, and correntropy is considered to be a statistical measure of the similarity between two random variables. This measure takes into account the second and the higher-order moments. Technically, the correntropy between X, Y is determined with the formula [26]

$$C(X,\ Y) = \mathrm{E}_{XY}\left[\kappa(X,\ Y)\right] =$$

$$= \iint \kappa(x,\ y) f_{XY}(x,\ y)\,\mathrm{d}x\mathrm{d}y,$$

where $\kappa(\cdot,\cdot)$ is some continuous positive defined function (a kernel); $f_{XY}(\cdot,\cdot)$ is the joint density function of the random variables X and Y. Most commonly, the Gaussian kernel of size $\sigma > 0$ is used:

$$\kappa(x,\ y) = G_\sigma(x - y) = \exp\left(\frac{-(x-y)^2}{2\sigma^2}\right).$$

In practice, the distribution of f_{XY} is usually unknown, thus the correntropy estimate $\hat{C}(X,\ Y)$ is used instead of $C(X,\ Y)$

$$\hat{C}(X,\ Y) = \frac{1}{N}\sum_{i=1}^{N} G_\sigma(x_i - y_i).$$

Note that the kernel size significantly affects the quality of correntropy filters. There are no actual general recommendations for the optimal selection of the parameter σ value that depends on the considered sample. This is the bottleneck of all the correntropy filters. Some adaptive techniques for determining the kernel size based on the update vector are given in [26, 32, 41], but the problem has not been solved yet, because generally the results are better when the kernel size is optimal.

Consider three correntropy modifications of the CD-CKF based on different maximum correntropy criteria.

The first modification named the CD-MCCKF-1 (Continuous-discrete maximum correntropy cubature Kalman filter) contains scalar parameters σ and δ. This modification has been obtained by replacing the equations of the correntropy discrete UKF from [30] with the relevant CD-CKF formulas. The following algorithm corresponds to this modification.

The CD-MCCKF-1 algorithm

Step 1. Repeat the step 1 of the CD-CKF algorithm.

Execute in a loop for $k = \overline{0,\ N-1}$:

Steps 2–5 are equivalent to the steps 2–5 of the CD-CKF algorithm.

Step 6. Obtain the matrix $\mathbf{H}(t_{k+1})$:

$$\mathbf{H}(t_{k+1}) = \mathbf{P}_{xy}^{\mathrm{T}}(t_{k+1} \mid t_k)\mathbf{P}^{-1}(t_{k+1} \mid t_k). \qquad (5)$$

Step 7. Find $\overline{\mathbf{S}}(t_{k+1})$:

$$\mathbf{S}_R(t_{k+1}) = \mathrm{Chol}\left[\mathbf{R}(t_{k+1})\right];$$

$$\overline{\mathbf{S}}(t_{k+1}) = \mathrm{diag}\left[\mathbf{S}(t_{k+1} \mid t_k),\ \mathbf{S}_R(t_{k+1})\right].$$

Step 8. Obtain $\mathbf{D}(t_{k+1})$ and $\mathbf{W}(t_{k+1})$:

$$\mathbf{D}(t_{k+1}) = \bar{\mathbf{S}}^{-1}(t_{k+1}) \begin{bmatrix} \hat{\mathbf{x}}(t_{k+1} \mid t_k) \\ \boldsymbol{\varepsilon}(t_{k+1}) + \mathbf{H}(t_{k+1})\hat{\mathbf{x}}(t_{k+1} \mid t_k) \end{bmatrix};$$

$$\mathbf{W}(t_{k+1}) = \bar{\mathbf{S}}^{-1}(t_{k+1}) \begin{bmatrix} \mathbf{I}_n \\ \mathbf{H}(t_{k+1}) \end{bmatrix}.$$

Step 9. Set $i = 1$ and evaluate

$$\hat{\mathbf{x}}^0(t_{k+1} \mid t_{k+1}) = \left[\mathbf{W}^{\mathrm{T}}(t_{k+1})\mathbf{W}(t_{k+1})\right]^{-1} \times$$
$$\times \mathbf{W}^{\mathrm{T}}(t_{k+1})\mathbf{D}(t_{k+1}).$$

Execute in a loop for i:

Step 10. Calculate

$$\mathbf{e}^i(t_{k+1}) = \mathbf{D}(t_{k+1}) - \mathbf{W}(t_{k+1})\hat{\mathbf{x}}^{i-1}(t_{k+1} \mid t_{k+1});$$

$$\mathbf{C}_x(t_{k+1}) = \mathrm{diag}\left[G_\sigma\left(e_1^i(t_{k+1})\right), \ldots, G_\sigma\left(e_n^i(t_{k+1})\right)\right];$$

$$\mathbf{C}_y(t_{k+1}) = \mathrm{diag}\left[G_\sigma\left(e_{n+1}^i(t_{k+1})\right), \ldots, G_\sigma\left(e_{n+m}^i(t_{k+1})\right)\right];$$

$$\mathbf{P}^i(t_{k+1} \mid t_k) = \mathbf{S}(t_{k+1} \mid t_k)\mathbf{C}_x^{-1}(t_{k+1})\mathbf{S}^{\mathrm{T}}(t_{k+1} \mid t_k);$$

$$\mathbf{R}^i(t_{k+1}) = \mathbf{S}_R(t_{k+1})\mathbf{C}_y^{-1}(t_{k+1})\mathbf{S}_R^{\mathrm{T}}(t_{k+1});$$

$$\mathbf{B}^i(t_{k+1}) = \mathbf{H}(t_{k+1})\mathbf{P}^i(t_{k+1} \mid t_k)\mathbf{H}^{\mathrm{T}}(t_{k+1}) + \mathbf{R}^i(t_{k+1});$$

$$\mathbf{K}^i(t_{k+1}) = \mathbf{P}^i(t_{k+1} \mid t_k)\mathbf{H}^{\mathrm{T}}(t_{k+1})\left[\mathbf{B}^i(t_{k+1})\right]^{-1};$$

$$\hat{\mathbf{x}}^i(t_{k+1} \mid t_{k+1}) = \hat{\mathbf{x}}(t_{k+1} \mid t_k) + \mathbf{K}^i(t_{k+1})\boldsymbol{\varepsilon}(t_{k+1}).$$

If $\dfrac{\left\|\hat{\mathbf{x}}^i(t_{k+1} \mid t_{k+1}) - \hat{\mathbf{x}}^{i-1}(t_{k+1} \mid t_{k+1})\right\|}{\left\|\hat{\mathbf{x}}^{i-1}(t_{k+1} \mid t_{k+1})\right\|} \le \delta$, then the end of the loop for i, else set $i = i + 1$.

Step 11. Define the filtering estimate and the corresponding error covariance matrix:

$$\hat{\mathbf{x}}(t_{k+1} \mid t_{k+1}) = \hat{\mathbf{x}}^i(t_{k+1} \mid t_{k+1});$$

$$\mathbf{P}(t_{k+1} \mid t_{k+1}) = \left[\mathbf{I}_n - \mathbf{K}^i(t_{k+1})\mathbf{H}(t_{k+1})\right]\mathbf{P}^i(t_{k+1} \mid t_k).$$

End of the loop for k.

The next algorithm corresponds to the second modification named the CD-MCCKF-2 and obtained by applying the CKF equations from [31] to the continuous-discrete case. This modification contains a scalar parameter σ.

The CD-MCCKF-2 algorithm

Step 1. Repeat step 1 of the CD-CKF algorithm.

Execute in a loop for $k = \overline{0, N-1}$:

Steps 2–5 are equivalent to the steps 2–5 of the CD-CKF algorithm.

Step 6. Calculate the measurement noise covariance matrix estimate:

$$\mathbf{S}_R(t_{k+1}) = \mathrm{Chol}\left[\mathbf{R}(t_{k+1})\right];$$

$$\mathbf{e}(t_{k+1}) = \mathbf{S}_R^{-1}(t_{k+1}) \times$$
$$\times \left[\mathbf{y}(t_{k+1}) - \mathbf{h}\left[\hat{\mathbf{x}}(t_{k+1} \mid t_k), \mathbf{u}(t_{k+1}), t_{k+1}\right]\right];$$

$$\mathbf{C}_y(t_{k+1}) = \mathrm{diag}\left[G_\sigma(e_1(t_{k+1})), \ldots, G_\sigma(e_m(t_{k+1}))\right];$$

$$\hat{\mathbf{R}}(t_{k+1}) = \mathbf{S}_R(t_{k+1})\mathbf{C}_y^{-1}(t_{k+1})\mathbf{S}_R^T(t_{k+1}).$$

Step 7. Define $\mathbf{P}_{yy}(t_{k+1} \mid t_k)$ with the expression (1) replacing $\mathbf{R}(t_{k+1})$ with $\hat{\mathbf{R}}(t_{k+1})$.

Steps 8–10. Carry out the steps 7–9 of the CD-CKF algorithm.

End of the loop for k.

The last algorithm corresponds to the third modification named the CD-MCCKF-3 and obtained by replacing the correntropy discrete UKF formulas from [32] with the relevant CD-CKF equations. This modification contains a scalar parameter σ.

The CD-MCCKF-3 algorithm

Step 1. Repeat the step 1 of the CD-CKF algorithm.

Execute in a loop for $k = \overline{0, N-1}$:

Steps 2–6 are equivalent to the steps 2–6 of the CD-CKF algorithm.

Step 7. Calculate the matrix $\mathbf{H}(t_{k+1})$ using the expression (5).

Step 8. Calculate the measurement noise covariance matrix estimate:

$$\hat{\mathbf{R}}(t_{k+1}) = \mathbf{P}_{yy}(t_{k+1} \mid t_k) - \mathbf{H}(t_{k+1})\mathbf{P}(t_{k+1} \mid t_k)\mathbf{H}^T(t_{k+1}).$$

Step 9. Find the scalar value $L(t_{k+1})$:

$$L(t_{k+1}) = G_\sigma\left(\left[\boldsymbol{\varepsilon}^T(t_{k+1})\hat{\mathbf{R}}^{-1}(t_{k+1})\boldsymbol{\varepsilon}(t_{k+1})\right]^{1/2}\right).$$

Step 10. Calculate the Kalman gain factor $\mathbf{K}(t_{k+1})$:

$$\mathbf{B}(t_{k+1}) = \hat{\mathbf{R}}(t_{k+1}) +$$
$$+ L(t_{k+1})\mathbf{H}(t_{k+1})\mathbf{P}(t_{k+1} \mid t_k)\mathbf{H}^{\mathrm{T}}(t_{k+1});$$

$$\mathbf{K}(t_{k+1}) = L(t_{k+1})\mathbf{P}(t_{k+1} \mid t_k)\mathbf{H}^{\mathrm{T}}(t_{k+1})\mathbf{B}^{-1}(t_{k+1}).$$

Step 11. Obtain the filtering estimate by repeating the step 8 of the CD-CKF algorithm.

Step 12. Calculate the error covariance matrix:

$$\mathbf{P}(t_{k+1} \mid t_{k+1}) = \left[\mathbf{I}_n - \mathbf{K}(t_{k+1})\mathbf{H}(t_{k+1})\right]\mathbf{P}(t_{k+1} \mid t_k).$$

End of the loop for k.

Comparison of the CD-CKF robust modifications

This section presents the comparison of the considered CD-CKF modifications effectiveness. The analysis has been made for the problem of tracking a space vehicle, entering the atmosphere, given in [10, 42]. In this case the state and the measurement equations in the planar Earth-centered Cartesian coordinate system are defined as follows

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} = \begin{bmatrix} x_3(t) \\ x_4(t) \\ D(t)x_3(t)+G(t)x_1(t)+w_1(t) \\ D(t)x_4(t)+G(t)x_2(t)+w_2(t) \\ w_3(t) \end{bmatrix},$$

$$t \in [t_0, t_N],$$

$$\begin{bmatrix} y_1(t_{k+1}) \\ y_2(t_{k+1}) \end{bmatrix} = \begin{bmatrix} r(t_{k+1})+v_1(t_{k+1}) \\ \theta(t_{k+1})+v_2(t_{k+1}) \end{bmatrix}, \quad k=\overline{0,\ N-1}.$$

Here $x_1(t)$, $x_2(t)$ are the space vehicle coordinates; $x_3(t)$, $x_4(t)$ are the corresponding coordinate velocities; $x_5(t)$ is the parameter of the vehicle aerodynamic properties; $y_1(t_{k+1})$ is the distance to the radar; $y_2(t_{k+1})$ is the angle between the space vehicle and the horizontal axis;

$$D(t) = b(t)\exp\left(\frac{6374 - R(t)}{13.406}\right)V(t);$$

$$b(t) = -0.59783\exp(x_5(t));$$

$$R(t) = \sqrt{x_1^2(t)+x_2^2(t)};$$

$$V(t) = \sqrt{x_3^2(t)+x_4^2(t)};$$

$$G(t) = -\frac{398600}{R^3(t)};$$

$$r(t_{k+1}) = \sqrt{(x_1(t_{k+1})-6374)^2 + x_2^2(t_{k+1})};$$

$$\theta(t_{k+1}) = \operatorname{arctg}\left(\frac{x_2(t_{k+1})}{x_1(t_{k+1})-6374}\right).$$

Let $t_0 = 0$, $N = 150$, $t_{k+1} = 0.1(k+1)$ and assume that all priori assumptions made in the "Structural-probabilistic description of the model" section are valid, and the statistical characteristics of the noise and the initial state are defined as follows

$$\mathbf{Q} = \operatorname{diag}\left[2.4064\times10^{-4},\ 2.4064\times10^{-4},\ 0\right],$$

$$\mathbf{R} = \operatorname{diag}\left[1,\ (0.017)^2\right],$$

$$\bar{\mathbf{x}}(t_0) = \begin{bmatrix} 6500.4 \\ 349.14 \\ -1.8093 \\ -6.7967 \\ 0.6932 \end{bmatrix},$$

$$\mathbf{P}(t_0) = \operatorname{diag}\left[10^{-6},\ 10^{-6},\ 10^{-6},\ 10^{-6},\ 1\right].$$

It is worth to be mentioned that all the considered robust modifications of the CD-CKF involve using certain parameters. The optimal values of these parameters (except for $\delta_0 = 10^{-8}$ in the CD-MCCKF-1) should be found individually for each run by minimizing the accumulated root mean square error (ARMSE) on some grid.

Using the presented CD-CKF modifications, we have processed data with the *stochastic* ordering of outliers. The data have been modeled in such a way they have 20 percent noise and the outliers noise variance equal to $\mathbf{R}_A = 10000\mathbf{R}$. Data modeling was performed using software developed in the Matlab system under the assumption that outliers are uniformly distributed over the entire modeling interval at the same time points for both observation components.

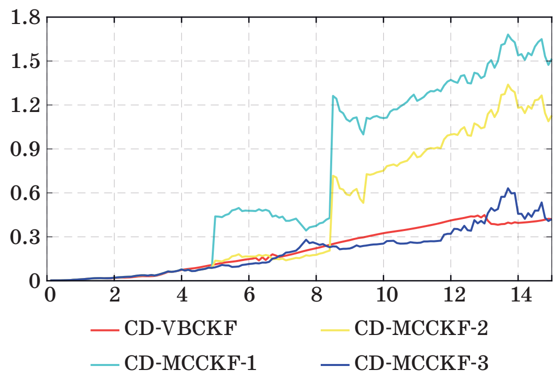
Figure 1 shows the dependence of the root mean square error (RMSE) on time.

Root mean square error value for each time point can be calculated using the equation

$$\mathrm{RMSE}(t_{k+1}) = \sqrt{\sum_{i=1}^{n}\left(x_i(t_{k+1}) - \hat{x}_i(t_{k+1} \mid t_{k+1})\right)^2},$$

$$k = \overline{0,\ N-1}.$$

In order to reduce the impact of the observed data on the estimation results, we have modeled $M = 100$ different samples. The filtering quality has been estimated based on the ARMSE value defined in accordance with the formula [33]



■ ***Fig. 1.*** The RMSE values for the stochastic outliers

■ ***Table 1.*** The values of the accumulated root mean square errors for the stochastic outliers

Filters	$ARMSE_1$	$ARMSE_2$	$ARMSE_3$	$ARMSE_4$	$ARMSE_5$	ARMSE
CD-VBCKF	0.191	0.161	0.039	0.027	1.114	1.143
CD-MCCKF-1	0.674	0.795	0.094	0.109	1.245	1.630
CD-MCCKF-2	0.579	0.579	0.082	0.082	1.211	1.466
CD-MCCKF-3	0.214	0.150	0.041	0.029	1.112	1.144

$$\mathrm{ARMSE} = \sqrt{\sum_{i=1}^{n} \mathrm{ARMSE}_i^2},$$

where

$$\mathrm{ARMSE}_i = \sqrt{\frac{1}{MN}\sum_{j=1}^{M}\sum_{k=0}^{N-1}\left(x_i^j(t_{k+1}) - \hat{x}_i^j(t_{k+1} \mid t_{k+1})\right)^2};$$

$x_i^j(t_{k+1})$ and $\hat{x}_i^j(t_{k+1} \mid t_{k+1})$ are the i-th components of the state vector and its filtering estimate for the j-th run.

The values of the accumulated root mean square errors for the various robust CD-CKF modifications are shown in the Table 1.

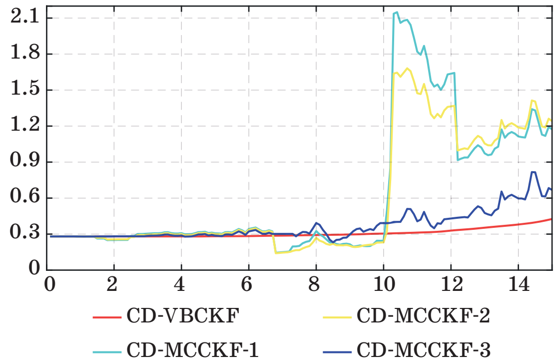
It should be emphasized that the presented data on a qualitative level repeat the results of the research carried out by the authors in [33] for a model of an underdamped oscillatory circuit.

We have also applied the presented CD-CKF modifications to data with *grouped* outliers. The outliers were organized in five groups containing six observations each. The variance of the outliers' noise was considered to be the same. The location of outlier's groups in the modeling interval is uniform and random.

Figure 2 illustrates the dependence of the RMSE on time.

One hundred different runs of the system have been made again. The aggregated values of the accumulated root mean square error are shown in the Table 2.

Hence, the CD-VBCKF and the CD-MCCKF-3 were also the most resistant modifications to the presence of grouped outliers.



■ ***Fig. 2.*** The RMSE values for the grouped outliers

Conclusion

In the paper, four distributional-robust modifications of the continuous-discrete cubature Kalman filter have been proposed. The study of the effectiveness of these modifications has been made for the problem of tracking a space vehicle during its reentry into the atmosphere. Two types of the outliers' ordering have been considered. The first one is the stochastic ordering, and the other one is the grouped ordering.

It has been found that the CD-VBCKF and the CD-MCCKF-3 provide the best results that have approximately equal qualities of estimation. Since the first filter requires finding the optimal values of four parameters (one of them is a matrix) to obtain the proper results, the second one requires estimating the only one parameter, so it seems to be appropriate to recommend the CD-MCCKF-3 for practical using.

It is further planned to modify CD-MCCKF-3 to provide computational robustness by developing a corresponding square-root modification.

■ ***Table 2.*** The values of the accumulated root mean square errors for the grouped outliers

Filters	$ARMSE_1$	$ARMSE_2$	$ARMSE_3$	$ARMSE_4$	$ARMSE_5$	ARMSE
CD-VBCKF	0.225	0.159	0.045	0.028	1.121	1.156
CD-MCCKF-1	0.806	1.049	0.122	0.133	1.451	1.972
CD-MCCKF-2	0.657	0.744	0.099	0.097	1.329	1.665
CD-MCCKF-3	0.224	0.162	0.046	0.028	1.125	1.160

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Исследование эффективности робастных к аномальным наблюдениям модификаций непрерывно-дискретного кубатурного фильтра Калмана

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Введение: характерное для практики присутствие в экспериментальных данных выбросов — аномальных наблюдений — способно существенно повлиять на качество обработки указанных данных. Многие динамические процессы описываются стохастическими нелинейными уравнениями. Современные нелинейные фильтры, среди которых кубатурный фильтр Калмана заслуживает особого внимания, не способны эффективно обрабатывать данные с аномальными наблюдениями. Одним из возможных путей решения этой проблемы является применение так называемых робастных методов, устойчивых к наличию выбросов в измерительных данных. **Цель исследования:** выявить наиболее эффективные из современных перспективных робастных модификаций непрерывно-дискретного кубатурного фильтра Калмана и дать соответствующие рекомендации по их применению. **Результаты:** рассмотрены часто возникающие на практике ситуации, когда процесс протекает непрерывно, а данные наблюдений снимаются дискретно. На основе вариационного байесовского и коррентропийного робастных подходов к оцениванию параметров случайных процессов предложены четыре модификации непрерывно-дискретного кубатурного фильтра Калмана. Во всех модификациях присутствуют параметры, оптимальные значения которых зависят как от используемой математической модели, так и от конкретной реализации выборки. Эти значения определяются численно путем минимизации на некоторой сетке накопленной средней квадратичной ошибки. Проведено исследование эффективности предложенных робастных модификаций на примере задачи слежения за космическим аппаратом при его входе в атмосферу в условиях случайного и группированного характерах расположения аномальных наблюдений. Выявлены два наилучших фильтра с близким качеством оценивания. К практическому применению рекомендован коррентропийный фильтр, имеющий один настраиваемый параметр. **Практическая значимость:** выявленный наиболее эффективный робастный фильтр можно использовать при решении различных прикладных задач, связанных с идентификацией стохастических нелинейных непрерывно-дискретных систем.

Ключевые слова — нелинейная фильтрация, кубатурный фильтр Калмана, выбросы, критерий максимальной коррентропии, вариационное байесовское оценивание, стохастическая непрерывно-дискретная система.

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